Panel 1

Last Time

One curve can have multiple parametrizations.

Length of a curve: $s = \int_a^s ||\vec{r}'(t)||\,dt$

Smooth curve: $\vec{r}'(t) \neq \vec{0}$

$r_1(t) = \langle t, t^2 \rangle$ smooth $\langle 1, 2t \rangle$

$r_2(t) = \langle t^2, t^4 \rangle$ not smooth $\langle 2t, 4t^3 \rangle$

Unit tangent $T(t) = \dfrac{r'(t)}{||r'(t)||}$

Ex: $\langle t^2+t, t^3 \rangle \rightarrow r'(t) = \langle 2t+1, 3t^2 \rangle$ smooth!

Panel 2

(5) Two particles travel along the space curves

$r_1(t) = \langle t, t^2, t^3 \rangle$, $r_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle$

Do they collide? Do their paths intersect?

$r_1(t) = r_2(s) \Rightarrow$ $s \neq t$ intersect

$s = t$ collide

no solution not intersect or collide

Panel 3

$$\frac{d}{dt}(\vec{u}\cdot\vec{v}) = \underline{\vec{u}'\cdot\vec{v} + \vec{u}\cdot\vec{v}'}$$

$u = \langle u_1, u_2\rangle$, $v = \langle v_1, v_2\rangle$, $u_1, u_2, v_1, v_2 : \mathbb{R}\to\mathbb{R}$

$$\frac{d}{dt}(u\cdot v) = (u_1v_1 + u_2v_2)' =$$

$$= u_1'v_1 + u_1v_1' + u_2'v_2 + u_2v_2'$$

$$u'\cdot v + u\cdot v'$$

#

Panel 4

$r(t) = \langle 3\sin(t), 4t, 3\cos(t)\rangle$

5 units long

$$s = \int_a^b \|r'(t)\|\,dt =$$

$$= \int_a^s \sqrt{9\cos^2(t) + 16 + 9\sin^2(t)}\,dt$$

starts at $(0,0,3)$

$\Rightarrow t=0$

$$5 = \int_{t=0}^{s} 5\,dt = 5(s) = \underline{5s}$$

$\Rightarrow s=1$

after moving 5 units along the curve, you are at

$$r(1) = \langle 3\sin(1), 4, 3\cos(1)\rangle$$ #

Panel 5

$T(t)$ if $r(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$

$r'(t) = \langle \sqrt{2}, e^t, -e^{-t} \rangle$

$\|r'(t)\| = \sqrt{2 + e^{2t} + e^{2t}} =$

$= \sqrt{2}\sqrt{1 + e^{2t}}$

$T(t) = \left(\|r'(t)\| \right)^{-1} r'(t) =$

$\frac{1}{\sqrt{2 + 2e^{2t}}} \langle \sqrt{2}t, e^t, e^{-t} \rangle$

Panel 6

Name: ______________

Quiz #4

(1) Describe the curve $\vec{r}(t) = \langle 2\cos(t), t, 2\sin(t) \rangle$ and find its length when $t \in [0, 3\pi]$.

(2) Find $\vec{r}'(t)$ if $\vec{r}(t) = \langle t e^{t^2}, \frac{\ln(t)}{t}, \sqrt{1-t^2} \rangle$

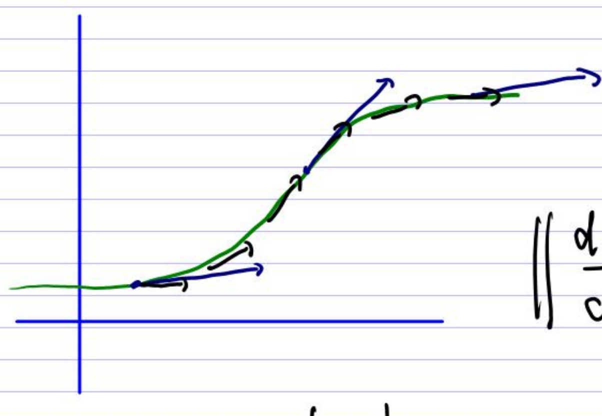
Panel 7

(3) Find the unit tangent $T(t)$ for the curve
$\vec{r}(t) = \langle 3\sin(t), 3\cos(t), 2t \rangle$

(4) Compute and sketch the tangent vector to the curve
$\vec{r}(t) = \langle t, t^3 \rangle$ when $t = 1$

Panel 8

Unit tangent give direction of change, normalized
$\Rightarrow$ Want to find rate of change of tangent:



special curve

$$\left\| \frac{dT}{dt} \right\| = \kappa$$

curvature!

Panel 9

We can now measure direction of curve (derivative) and length (arc length). Next we measure

Def: Curvature $\kappa = \frac{\|T'(t)\|}{\|r'(t)\|}$.

Panel 10

Ex: What is the curvature of a circle of radius R?

$$\vec{r}(t) = \langle R\cos(t), R\sin(t) \rangle$$

$$r'(t) = \langle -R\sin(t), R\cos(t) \rangle$$

$$\|r'\| = \sqrt{R^2(\sin^2 + \cos^2)} = R$$

$$T = \frac{1}{R}\vec{r}\,'(t) = \langle -\sin(t), \cos(t) \rangle$$

$$T' = \langle -\cos(t), -\sin(t) \rangle$$

$$\kappa = \frac{\|T'\|}{\|r'\|} = \frac{1}{R}$$