Panel 1

$P(1,1,1)$, $\ell = \langle t+1, 2-3t, 2t \rangle$

$d = \frac{\|\vec{QR} \times \vec{QP}\|}{\|QR\|}$ $Q(1,2,0)_{t=0}$

$R(2,-1,2)_{t=1}$

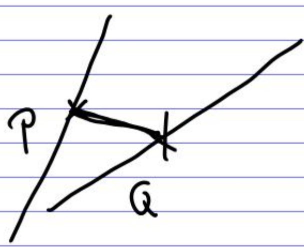
$QR = \langle 1,-3,2 \rangle$

$QP = \langle 0,-1,+1 \rangle$

$\frac{\|\langle 1,1,1 \rangle\|}{\|\langle 1,-3,2 \rangle\|} = \frac{\sqrt{3}}{\sqrt{14}}$

Panel 2

$\ell_1(t) = \langle t+1, 2-3t, 2t \rangle$, $\ell_2(t) = \langle 3-3t, 2+9t, 4-6t \rangle$



$u = \langle 1,-3,2 \rangle$

$v = \langle -3, 9, -6 \rangle$

$v \times w = 0$

$d = \|\text{proj}_{\vec{u}} \vec{PQ}\| = \frac{\|\vec{PQ} \times u\|}{\|u\|} = \frac{\langle 2,0,4 \rangle \cdot \langle 1,-3,2 \rangle}{\sqrt{14}}$

$P(1,2,0)$ $\vec{PQ} = \langle 2,0,4 \rangle$ $= \frac{10}{\sqrt{14}}$

$Q(3,2,4)$

Panel 3

Last Time: t = time

Vector-valued functions: $r(t) = \langle x(t), y(t), z(t) \rangle$

space curve

Derivatives $\frac{d}{dt}\vec{r}(t) =$

Limits $\lim_{t \to t_0} \vec{r}(t)$ boring

Integrals $\int_a^b \vec{r}(t)dt =$

~~Graphs of $\vec{r}(t)$~~ complicated — Maple / WA

Length of curve: $\ell = \int_a^s \|r'(t)\| dt$

Panel 4

Review Questions spiral: $\langle t\cos(t), t\sin(t), t \rangle$

① Describe the curves given by

a) $\vec{r}(t) = \langle \cos(t), t, \sin(t) \rangle$ slinky around y-axis ~~spiral~~

b) $\vec{r}(t) = \langle 5t, 1-t, 2t+1 \rangle$ line

② If $\vec{r}(t) = \langle t\sin(t), e^{t^2} \rangle$, find $\vec{r}'(t)$

$r'(t) = \langle \sin(t) + t\cos(t), 2te^{t^2} \rangle$

Panel 5

③ If $\vec{r}(t) = \left\langle \frac{\sin(t)}{t}, \frac{\cos(t)-1}{t}, \frac{e^{t^2}-1}{t^2} \right\rangle$

find $\lim_{t \to 0} \vec{r}(t)$

$\lim_{t \to 0} \left\langle 1, 0, \frac{2te^{t^2}}{2t} \to 1 \right\rangle$

④ Find the length of $\langle 2t, t, 3t-1 \rangle$ for $0 \le t \le 1$

$\ell = \int_0^1 \|r'(t)\| \, dt = \int_0^1 \| 2, 1, 3 \| \, dt = \sqrt{14} \int_0^1 dt = \sqrt{14}$

Panel 6

Length of $\langle t\cos(t), t, t\sin(t) \rangle$, 0 to π

$\ell = \int_0^\pi \|$

$r'(t) = \langle \cos(t) - t\sin(t), 1, \sin(t) + t\cos(t) \rangle$

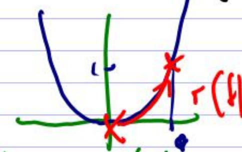
$\ell = \int_0^\pi \sqrt{(\cos(t) - t\sin(t))^2 + 1 + (\sin(t) + t\cos(t))^2} \, dt =$

WA

Panel 7

It is possible for one curve to have many different parametrizations:

Ex: $r_1(t) = \langle t, t^2 \rangle$, $t = 0, 1$



$x = t,\ y = t^2 \rightarrow y = x^2$ parabola

Same as: $r(t) = \langle t^3, t^6 \rangle$, $t \in 0, 1$

Same as: $r(t) = \langle 2t, 4t^2 \rangle$, $t \in (0, \frac{1}{2})$

Note: $f(x) = \sin(x) + x^2$ as a curve

$r(t) = \langle t, \sin(t) + t^2 \rangle$

Panel 8

Find length of curve $r(t) = \langle 1+t, 2+2t, 3+3t \rangle$ for $t \in [0, 1]$. Replace t by $2t$ and make sure to cover the same line segment $r(0) = \langle 1, 2, 3 \rangle$, $r(1) = \langle 2, 4, 6 \rangle$

a) Interpret the effect of this parameter change

$r(t) = \langle 1 + (2t), 2 + 2(2t), 3 + 3(2t) \rangle$, $t \in [0, \quad]$

$r(0) = \langle 1, 2, 3 \rangle$, $r(\frac{1}{2}) = \langle 2, 4, 6 \rangle$

b) Find the length of this "new" and compare.

$r_1(t) = \langle 1+t, 2+2t, 3+3t \rangle$, $\quad l = \int_0^1 \| \langle 1, 2, 3 \rangle \| dt = \underline{\sqrt{14}}$

$r_2(t) = \langle 1+2t, 2+4t, 3+6t \rangle$ $\quad l = \int_0^{1/2} \| \langle 2, 4, 6 \rangle \| dt = \frac{1}{2}\sqrt{56} = \frac{1}{2} 2\sqrt{14}$

Panel 9

Ex: Compute length of $r(t) = \langle t, \sqrt{1-t^2} \rangle$, $t = -1$ to 1

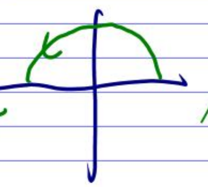
$$r'(t) = \left\langle 1, \tfrac{1}{\cancel{2}}(1-t^2)^{-1/2} \cdot (-\cancel{2}t) \right\rangle = \left\langle 1, \frac{-t}{\sqrt{1-t^2}} \right\rangle$$

$$\ell = \int_{-1}^{1} \|r'(t)\| \, dt = \int_{-1}^{1} \sqrt{1 + \frac{t^2}{1-t^2}} \, dt =$$

$$= \int_{-1}^{1} \sqrt{\frac{1-\cancel{t^2}+\cancel{t^2}}{1-t^2}} \, dt = \int_{-1}^{1} \frac{1}{\sqrt{1-t^2}} \, dt = \arcsin(t)\Big|_{-1}^{1} = \pi$$

$x = t$, $y = \sqrt{1-t^2} = \sqrt{1-x^2} \Rightarrow y^2 = 1 - x^2$, $x^2 + y^2 = 1$

$r(t) = \langle \cos(t), \sin(t) \rangle$, $t \in [0, \pi] \Rightarrow \int_0^{\pi} \|r'\| \, dt = \pi$

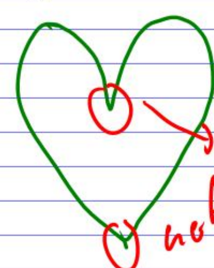


Panel 10

Def: A curve $r(t)$ is called smooth if $r'(t) \neq \vec{0}$, i.e. if the components of r' are not simultaneously zero.

Def: If $r(t)$ is a smooth curve then

$$T(t) = \frac{r'(t)}{\|r'(t)\|} \quad \text{unit tangent}$$



not smooth

Ex: $r(t) = \langle t, t^2 \rangle$ find $T(t)$.

$r(t) = \langle t, t^2 \rangle$

$r'(t) = \langle 1, 2t \rangle$

$$T(t) = \frac{1}{\sqrt{1+4t^2}} \langle 1, 2t \rangle$$