Panel 1

Last Time

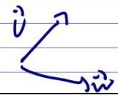
Line in $\mathbb{R}^n$: $\ell(t) = P_0 + t\vec{v}$ ← parametric eqn.

P_0 a point on the line, $\vec{v}$ is direction of line

Plane in $\mathbb{R}^3$: $ax + by + cz + d = 0$ ← scalar eqn.

$\vec{n} = \langle a, b, c \rangle$ normal to plane, i.e. any vector in the plane is perp. to $\vec{n}$

alt: $p(s,t) = P_0 + \vec{v}t + \vec{w}s$



Panel 2

Intersections:

$= \langle 1+t, 2+t, 3+t \rangle$

Find intersection of $\ell(t) = \langle 1,2,3 \rangle + t\langle 1,1,1 \rangle$ and $2x - y + z = 0$

$2(1+t) - (2+t) + (3+t) = 3 + 2t = 0 \Rightarrow t = -\frac{3}{2}$

$\boxed{\ell(-\tfrac{3}{2}) = \langle -\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{3}{2} \rangle}$

Find intersection of $\ell_1(t) = \langle 0,1,0 \rangle + t\langle 1,0,1 \rangle$ and

$\ell_2(s) = \langle -1,-2,1 \rangle + t\langle 2,3,0 \rangle$

$\ell_1(t) = \ell_2(s)$ solve for t, s, and subst. into the 3rd eqn.

Find intersection of $-x + y + z = 0$ and

$2x - y + z = 1$

Panel 3

$P(1,2,3)$, $Q(-1,-1,2)$, $R(5,4,1)$

line between $\vec{PQ}$

$\ell(t) = (1,2,3) + t(-2,-3,-1)$

plane through P, Q, R

$\vec{PQ} = \langle -2,-3,-1 \rangle$

$\vec{PR} = \langle 4,2,-2 \rangle$

$$\begin{vmatrix} i & j & k \\ -2 & -3 & -1 \\ 4 & 2 & -2 \end{vmatrix} = \langle -4, +4, -4 \rangle$$

$\hat{n} = \langle 1, -1, 1 \rangle$, $x - y + z + d = 0$

$1 - 2 + 3 + d = 0 \Rightarrow d = 0$

Panel 4

$(A,0,0)$, $(0,B,0)$, $(0,0,C)$

$\vec{v} = (-A, B, 0)$

$\vec{w} = (-A, 0, C)$

$$\begin{vmatrix} i & j & k \\ -A & B & 0 \\ -A & 0 & C \end{vmatrix} = \langle BC, AC, AB \rangle$$

$BCx + ACy + ABz + d = 0$, $d = -ABC$

Panel 5

$x + y + z = 0$ and $2x - y + z = 1$

Panel 6

Quiz #3

Name ________

(1) Find equations for:

a) a line through $P(3,-1,2)$ and $Q(0,1,1)$

b) a plane through $P(3,-1,2)$, $Q(0,1,1)$, $R(-1,1,2)$

Panel 7

(2) Is $P(0,1,2)$ on line $\ell(t) = \langle 2,3,0\rangle + t\langle -1,-1,1\rangle$?

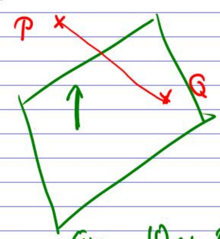
(4) Are $2x - y + 3z = 5$ and $3x + 6y - z = 0$ perpend.?

(5) Find intersection of $\ell(t) = \langle 1,1,2\rangle + t\langle 2,-1,3\rangle$ and $2x - y + 3z = 21$

Panel 8

Distances

a) Distance between point P and plane $ax + by + cz + d = 0$



1. Find Q on the plane
2. Find $\vec{PQ}$
3. $d = \left\| \text{proj}_{\vec{n}} \vec{PQ} \right\| = \dfrac{|\vec{PQ}\cdot \vec{n}|}{\|\vec{n}\|}$

Ex: $10x + 2y - 2z = 5$ distance to origin $(0,0,0)$

1) $Q\left(\frac{1}{2}, 0, 0\right)$

$\vec{PQ} = \left\langle -\frac{1}{2}, 0, 0\right\rangle$

or $Q\left(0, \frac{5}{2}, 0\right)$

$d = \dfrac{|\vec{PQ}\cdot \vec{n}|}{\|\vec{n}\|} = \dfrac{\left|\left\langle -\frac{1}{2}, 0, 0\right\rangle \cdot \langle 10, 2, -2\rangle\right|}{\sqrt{108}}$

$= 5/\sqrt{108}$

Panel 9

Ex: Find distance of $10x + 2y - 2z = 5$ to origin

1. Find any Q in the plane
2. Find $\vec{PQ}$
3. $d = \|\text{proj}_{\vec{n}} \vec{PQ}\| = 5/\sqrt{108}$

Panel 10

Find distance between

a) $10x + 2y - 2z = 5$ and $P(1,1,1)$

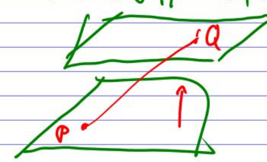
b) $10x + 2y - 2z = 5$ and $x + y + z = 1$

c) $10x + 2y - 2z = 5$ and $5x + y - z = 1$

Dist. between planes: parallel?

$(10,2,-2) \parallel \langle 1,1,1)$ No! $\Rightarrow d = 0$

$\langle 10,2,-2) \parallel \langle 5,1,-1)$ Yes!



P any point on plane 1

Q any point on plane 2

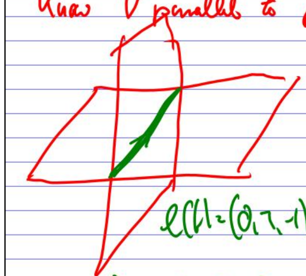
$d = \|\text{proj}_{\vec{n}} \vec{PQ}\| = \dfrac{|\vec{PQ}\cdot\vec{n}|}{\|n\|}$

Panel 11

We know the planes $10x + 2y - 2z = 6$ and $x + y + z = 1$ are not parallel. Thus, they intersect! Find intersection!

They intersect in a line $\ell(t) = P_0 + \vec{v}\, t$

Know $\vec{v}$ parallel to plane 1 and to plane 2



$\Rightarrow \vec{v}$ is perp to n_1 and to n_2

$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 10 & 2 & -2 \\ 1 & 1 & 1 \end{vmatrix} = \langle 4,-12,8\rangle$

$\ell(t) = (0,2,-1) + t(1,-3,2)$

$\vec{v} = \langle 1,-3,2\rangle$

pick one coord of line to be zero: $x = 0$: $2y - 2z = 6$, $y + z = 1$, $4y = 8$

$y = 2$, $z = -1$

Panel 12

Formula. The distance between $P(x_0, y_0, z_0)$ and plane $ax + by + cz + d = 0$ is $d =$

HW

Panel 13

Summary: Distance between

$P_0(x_0, y_0) \in \mathbb{R}^2$ and line $ax+by+c=0$ in $\mathbb{R}^2$

$$d = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$$ (every book)

$P_0(x_0, y_0, z_0) \in \mathbb{R}^3$ and plane $ax+by+cz+d=0$ in $\mathbb{R}^3$

$$d = \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$$ what if P on plane?

Distance between 2 planes ✓

plane + line? as above

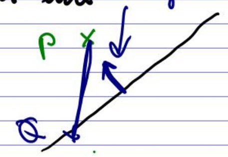
⇒ 2 lines, point + line in $\mathbb{R}^3$ ⇐

Panel 14

Distance of point $P(x_0, y_0)$ from line easy in $\mathbb{R}^2$

$ax+by+c=0$ in $\mathbb{R}^2$:

$x+y+1=0$ or $\ell(t) = (0,-1) + (1,-1)t$



Distance of point $P(x_0, y_0, z_0)$ from line through R and Q in $\mathbb{R}^3$: