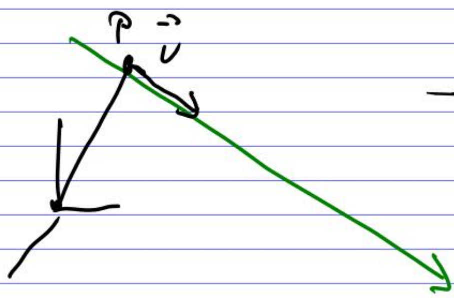
Panel 1

Last time

Dot Product: ✓
Cross Product: ✓

Parametric Equation of line:

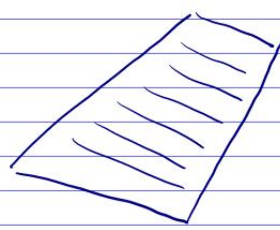


$$\ell(t) = P + \vec{v}\,t$$

$$= \langle p_1 + tv_1,\ p_2 + tv_2,\ p_3 + tv_3 \rangle$$

Panel 2

Planes in $\mathbb{R}^3$

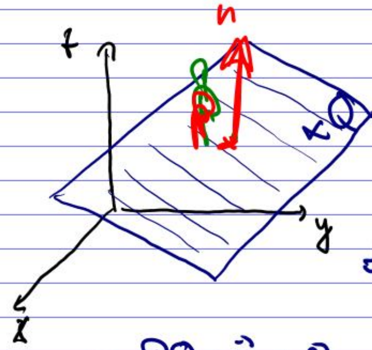


A plane in $\mathbb{R}^3$ is uniquely determined by

- three points
- two vectors + one point
- one vector normal (perpendicular) to the plane + one point.

Panel 3

Suppose a plane is normal to $\vec{n} = \langle a, b, c \rangle$ and goes through $P(x_0, y_0, z_0)$.



Take a point $Q(x, y, z)$ on the plane

$\Rightarrow$ the vector $\vec{PQ}$ is on the plane

$\Rightarrow PQ \cdot \vec{n} = 0$

$\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ is the (scalar) equation of the plane.

Panel 4

Def: The equation of a plane with normal vector $\vec{n} = \langle a, b, c \rangle$ through the point $P_0(x_0, y_0, z_0)$ is:

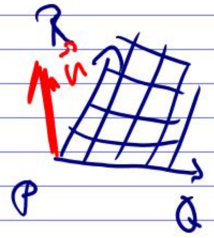
$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$\Leftrightarrow ax + by + cz + d = 0$

Panel 5

<u>Scalar</u> <u>equation</u> of Plane through $P(x_0, y_0, z_0)$ with normal vector $\vec{n} = \langle a, b, c\rangle$ is $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

<u>Ex</u>: Plane through $P(1,3,2)$, $Q(3,-1,6)$ and $R(5,2,0)$



$\langle 1,-2,2\rangle$

$\vec{PQ} = \langle 2,-4,4\rangle$, $\vec{PR} = \langle 4,-1,-2\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 4 & -1 & -2 \end{vmatrix} = \langle 4+2, (-2-8), -1+8\rangle = \langle 6, 10, 7\rangle$$

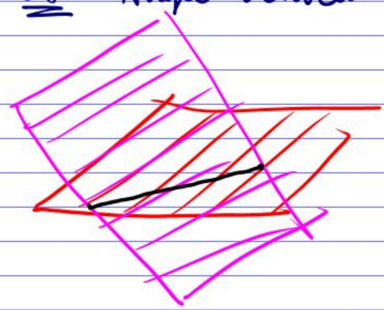
$\Rightarrow 6(x-1)+10(y-3)+7(z-2)=0$

$6x+10y+7z-6-30-14=0 \Rightarrow \underline{6x+10y+7z=50}$

Panel 6

<u>Scalar</u> <u>equation</u> of Plane through $P(x_0, y_0, z_0)$ with normal vector $\vec{n} = \langle a, b, c\rangle$ is $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

<u>Ex</u>: Angle between planes $x+y+z=1$ and $x-2y+3z=1$



$\vec{n}_1 = \langle 1,1,1\rangle$

$\vec{n}_2 = \langle 1,-2,3\rangle$

$$\cos(\theta) = \frac{\langle 1,1,1\rangle \cdot \langle 1,-2,3\rangle}{\sqrt{3}\,\sqrt{14}} = \frac{1-2+3}{\sqrt{3\cdot 14}} = \underline{\frac{2}{\sqrt{42}}}$$

Panel 7

Consider the plane $2x + 3y + 4z = 12$

a) Find any point on plane $(0,0,3)$ or $(0,4,0)$

b) Find point with $x = 42$, $y = 7$ on the plane

$\left(42, 7, z = \frac{1}{4}(12 - 3\cdot 7 - 2\cdot 42)\right)$

c) Is $P(0,2,-1)$ on the plane ?

$2\cdot 0 + 3\cdot 2 + 4\cdot(-1) \neq 12$ No

d) Does $\ell_1(t) = \langle 1,2,0\rangle + t\langle 3,-1,1\rangle$ intersect plane? How about $\ell_2(t) = \langle 0,4,0\rangle + t\langle 2,-4,2\rangle$?

$\ell_1(t) = \langle 1+3t, 2-t, t\rangle$: $2(1+3t) + 3(2-t) + 4t =$

$2 + 6t + 6 - 3t + 4t = 12 \Rightarrow t =$

Panel 8

$\ell_1(t) = \langle 1,2,0\rangle + t\langle 3,-1,1\rangle$ intersect

$2x + 3y + 4z = 12$

$\langle 2,3,4\rangle \cdot \langle 2,-4,2\rangle$

$= 0$ ✓

$\ell_1(t) = \langle 1+3t, 2-t, t\rangle$

$2(1+3t) + 3(2-t) + 4(t) = 12$

$2 + 6t + 6 - 3t + 4t = 12$

$8 + 7t = 12$

$7t = 4$

$\underline{\underline{t = \frac{4}{7}}}$ $\Rightarrow$ point of intersection is

$\ell\left(\frac{4}{7}\right) = \langle \;,\;,\;\rangle$

Panel 9

$\langle 2t, 4-4t, 2t \rangle$

$\ell_2(t) = \langle 0,4,0 \rangle + t\langle 2,-4,2 \rangle$ intersect

$2x + 3y + 4z = 12$

1) $\langle 2,-4,2 \rangle \cdot \langle 2,3,4 \rangle = 0 \Rightarrow$ line parallel to plane!
2) Point $(0,4,0)$ is also on plane
$\Rightarrow$ line on the plane

Alternate: $2(2t) + 3(4-4t) + 4(2t) = 12$

$\cancel{4t} + 12 - \cancel{12t} + \cancel{8t} = 12$

$12 = 12$

Panel 10

Graphing Planes: Planes can be visualized by looking at the traces in the coordinate planes.

$x + y + z = 1$

$z = 1 - x$

xz-plane $(y=0)$: $x + z = 1$

zy-plane $(x=0)$: $y + z = 1$

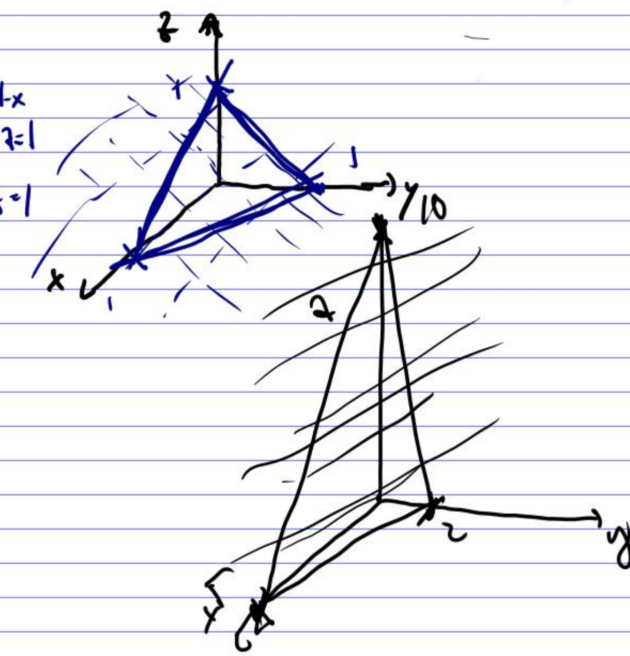
$2x + 5y + z = 10$

$y = 0 = z$: $x = 5$

$x = y = 0$: $z = 10$

$x = z = 0$: $y = 2$

$3x + y + 2z = 6$



Panel 11

Intersections:

$= \langle 1+t, 2+t, 3+t \rangle$ (x, y, z)

Find intersection of $\ell(t) = \langle 1,2,3 \rangle + t\langle 1,1,1 \rangle$ and

HW $\quad 2x - y + z = 0$

$t_0 = \# \Rightarrow \ell(t_0)$

Find intersection of $\ell_1(t) = \langle 0,1,0 \rangle + t\langle 1,0,1 \rangle$ and

$\ell_2(s) = \langle -1,-2,1 \rangle + t\langle 2,3,0 \rangle$

$\ell_1(t) = \ell_2(s)$

$0 + t = -1 + 2s \qquad 1 = -1 + 2 \cdot 1$ checks out ✓

$1 = -2 + 3s \Rightarrow \underline{\underline{s = 1}}$ intersect at

$\underline{\underline{t = 1}}$ ✓

$P(1,1,1)$

Panel 12

$x + y + z = 0$ and $2x - y + z = 1$

1.) Check if they intersect at all:

If they were parallel, they won't intersect

$n_1 = \langle 1,1,1 \rangle \quad n_2 = \langle 2,-1,1 \rangle$

$c\, n_1 = n_2$ No! $\Rightarrow$ Not parallel

They must intersect in a line.

This line must go through (xy)-plane $\Rightarrow \underline{\underline{z = 0}}$

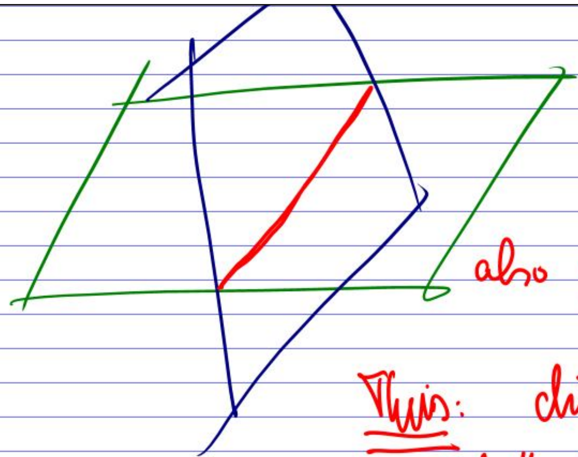
Also. $x + y + \cancel{z} = 0 \qquad 3x = 1, \quad \underline{\underline{x = 1/3}}$

$2x - y + \cancel{z} = 1$

$\underline{\underline{y = -1/3}}$

$P(\frac{1}{3}, -\frac{1}{3}, 0)$ is one line of intersection

Panel 13



line is on plane I

(dir of line) (normal$_1$) = 0

also (dir of line) · (normal$_2$) = 0

__Thus:__ dir. vector is perp. to both normal vectors

$\Rightarrow$ dir vector is parallel to $\underline{n_1 \times n_2}$

$\Rightarrow \vec{v} = n_1 \times n_2$ $\qquad \ell = P + t\vec{v}$

$P = (0, \frac{1}{3}, -\frac{1}{3})$