Panel 1

Last Time

Dot Product. $\vec{a} \cdot \vec{b} = \begin{cases} a_1b_1 + a_2b_2 + a_3b_3 \\ \|a\|\|b\| \cdot \cos(\theta) \end{cases}$

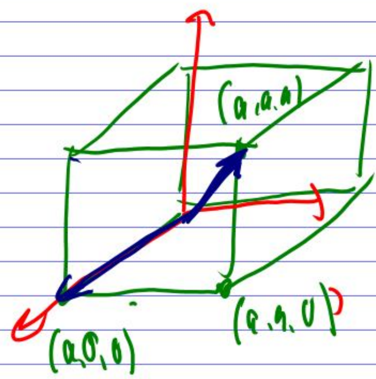
Projection of $\vec{b}$ onto $\vec{a}$: $\text{proj}_{\vec{a}}(\vec{b}) = \frac{a \cdot b}{\|\vec{a}\|^2} a$

Length of $\text{proj}_{\vec{a}}(\vec{b})$: $\text{comp}_{\vec{a}}(\vec{b}) = \frac{|a \cdot b|}{\|a\|}$

Cross Product. $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$

$\langle a_2b_3 - a_3b_2, -(a_1b_3 - a_3b_1), a_1b_2 - a_2b_1 \rangle$

Panel 2



length cube a

angle between

(a,a,a) and $(a,0,0)$:

$\cos(\theta) = \frac{(a,a,a) \cdot (a,0,0)}{\sqrt{3}a \cdot a} = \frac{a^2}{\sqrt{3}a^2}$

$= \underline{\underline{1/\sqrt{3}}}$

Panel 3

Quiz #2

Name: ______________

(1) Find the dot product $\langle 3,-2,1\rangle \cdot \langle 1,2,2\rangle$

(2) Which vector is perpendicular to $\langle 3,-2,1\rangle$:

a) $\langle 1,1,1\rangle$

b) $\langle 2,4,2\rangle$

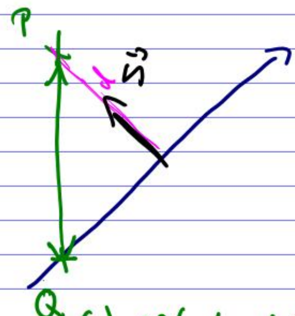
Panel 4

(3) Find the projection of $\langle 3,-1,-2\rangle$ onto $\langle 3,3,1\rangle$

(4) Find the cross product $\langle 3,-2,1\rangle \times \langle 1,2,2\rangle$

Panel 5

Find a formula for the distance between $P(x_0, y_0)$ and a line $ax + by + c = 0$.



(1) Take any Q on the line
$\rightarrow \vec{PQ} : \langle -x_0, -\frac{c}{b} - y_0 \rangle$
(2) Find vector $\vec{n}$ perp. to line

$\Rightarrow d = \| proj_{\vec{n}} (PQ) \|$

(a) Which direction is the line?

Q $①(0, -\frac{c}{b}) \in$ line and $②(-\frac{c}{a}, 0) \in$ line

direction of line: $②-①: \langle -\frac{c}{a}, \frac{c}{b} \rangle \cdot \frac{ab}{c} = \langle -b, a \rangle$

$\vec{n} = \langle a, b \rangle$ is perp (normal) to line $ax + by + c = 0$

Panel 6

$$d = \| proj_{\vec{n}} PQ \| = \left\| \frac{\langle -x_0, -y_0 - \frac{c}{b} \rangle \cdot \langle a, b \rangle}{\| \vec{n} \|^2} \vec{n} \right\|$$

$$= \frac{| -ax_0 - by_0 - c |}{\| n \|} = \frac{| ax_0 + by_0 + c |}{\sqrt{a^2 + b^2}}$$

memorize

$3x + 4y - 2 = 0$

Ex: dist. between $(1,1)$ and $3x + 4y = 2$

$\vec{n} = \langle 3, 4 \rangle$: $d = \frac{3 \cdot 1 + 4 \cdot 1 - 2}{\sqrt{25}} = \frac{5}{5} = 1$

Panel 7

<u>Dot product:</u> $\vec{a} \cdot \vec{b} = \|a\| \, \|b\| \cos(\theta)$

$\vec{a} \cdot \vec{b} = 0 \iff$ perp.

<u>Cross Product:</u> $\vec{a} \times \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \sin(\theta)$

$\vec{a} \times \vec{b} = 0 \iff$ parallel!

Recall: if $c \cdot \vec{a} = \vec{b}$ parallel $\vec{a}, \vec{b}$

Panel 8

Cross out the expressions that do not make sense. For the rest, is the answer a vector or a scalar?

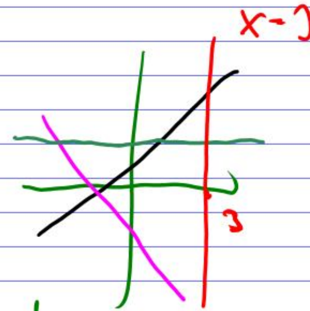
$a \cdot (b \times c)$ ✓	$(a \cdot b) \times c$ ✗
$(a \cdot b) \cdot c$ ✗	$(a \times b) + c$ ✓
$a \times (b \cdot c)$ ✗	$(a \cdot b) \times (c \cdot d)$ ✗
$\|a\| (b \cdot c)$ ✓	$(a \cdot b) + c$ ✗
$a \times (b \times c)$ ✓	$(a \times b) \cdot (c \times d)$ ✓
$a \cdot (b + c)$ ✓	$\|a\| (b \times c)$ ✓
$(a \times b) + c$ ✓	$\|a \times b\|$ ✓

Panel 9

Of Lines and Planes

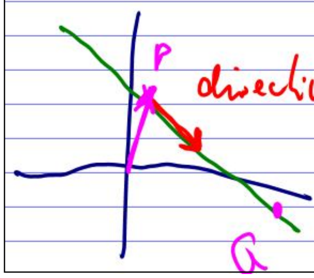
Line: $y = mx + b$

no good! one exception



does not generalize!

line in $\mathbb{R}^3$: $z = m_1 x + m_2 y + b$ ✗ not a line!



direction $\vec{v}$, P point on line.

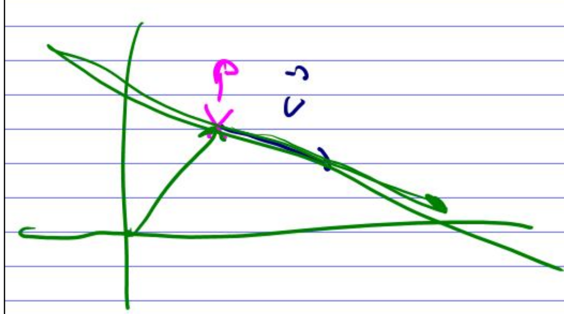
$Q = P + t\vec{v}$

Panel 10

Def: If $P(x_0, y_0, z_0)$ is a point on a line, and $\vec{v} = \langle a, b, c \rangle$ is the direction of the line, then the (parametric) equation of the line is:

$$\ell(t) = P + t\vec{v}$$

works in $\mathbb{R}^n$, for any n!



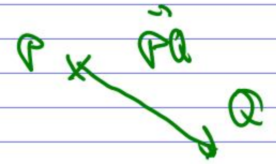
Panel 11

Ex: Find equation of a line

a) through $(5,1,3)$ and parallel to $\vec{v} = \langle 1,4,-2\rangle$

$$\ell(t) = \langle 5,1,3\rangle + t\langle 1,4,-2\rangle = \langle 5+t,\ 1+4t,\ 3-2t\rangle$$

b) through $P(1,2,3)$ and $Q(4,1,1)$



$$\vec{PQ} = \langle 3,-1,-2\rangle = (Q-P)$$

$$\ell(t) = \langle 1,2,3\rangle + t\langle 3,-1,-2\rangle$$

Panel 12

Ex: Line through $P(1,3)$ and $Q(3,2)$

(old-fashioned) $m = \dfrac{2-3}{3-1} = \dfrac{-1}{2} = -1/2$

$y = -\frac{1}{2}x + b$ HW

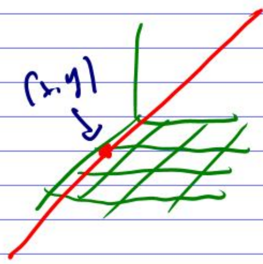
New way: $\ell(t) = \langle 1,3\rangle + t\langle 2,-1\rangle$

Line through $P(1,4)$ and $Q(1,5)$

$$\ell(t) = (1,4) + t(0,1) = \langle 1, 4+t\rangle$$

Panel 13

Ex: At what point does $\langle 2,4,-3\rangle + t\langle 1,-5,4\rangle = \ell(t)$ intersect the xy-plane



$\ell(t) = \langle 2+t, 4-5t, -3+4t\rangle$

$z=0 = -3+4t \Rightarrow t = 3/4$

$\ell(\tfrac{3}{4}) = \langle \underbrace{2+\tfrac{3}{4}}_{x}, \underbrace{4-5\tfrac{3}{4}}_{y}, 0\rangle$

Ex: Is $P(1,2,3)$ on line $\langle 2,4,-3\rangle + t\langle -1,-2,3\rangle$

If so: $(1,2,3) = \langle \underbrace{2-t}_{t=1}, \underbrace{4-2t}_{2}, \underbrace{-3+3t}_{0}\rangle$ $\boxed{NO}$

Find a) any point on $\langle 1,2,3\rangle + t\langle 2,-1,0\rangle$

$t=0$: $(1,2,3)$, $t=1$: $(3,1,3)$, ...

b) with x-comp. 42: ✓

Panel 14

Suppose 2 lines are $\ell_1(t) = \langle 1+t, -2+3t, 4-t\rangle$

$\ell_2(t) = \langle 2t, 3+t, -3+4t\rangle$

a) Are they parallel?

$\ell_1(t) = \langle 1,-2,4\rangle + t\langle 1,3,-1\rangle$ No, because $\langle 1,3,-1\rangle$

$\ell_2(t) = \langle 0,3,-3\rangle + t\langle 2,1,4\rangle$ and $\langle 2,1,4\rangle$ not parall

b) Do they intersect? If so:

$\ell_1(t) = \ell_2(s)$

(1) $1+t = 2s \Rightarrow t = 2s-1$

(2) $-2+3t = 3+s$ $\quad -2+3(2s-1) = 3+s$

(3) $4-t = -3+4s$ $\quad -2+6s-3 = 3+s$

$5s = 8$

$s = 8/5$

$t = \frac{16}{5} - 1 = \frac{11}{5}$

Do they even check in (3)