Panel 1

Last time

Coordinate system in $\mathbb{R}^3$

Distance formula between points

$$d=\sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$$

3D Objects

spheres (complete square)

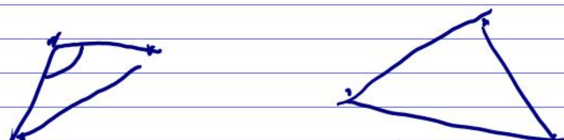
Vectors $\quad x^2+y^2=4 \quad , y=2x$

- length
- unit vector
- add, subtract

Panel 2

$\mathbb{R}^3$ - can't see

triangles



$(3,-2,-3)$

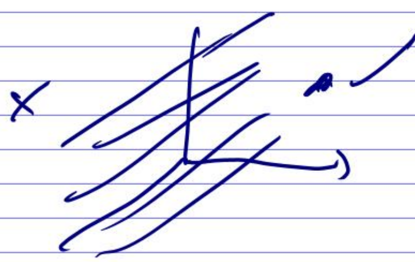
$(7,0,1)$

$(1,2,1)$

$D=\sqrt{4^2+4+4^2}=\sqrt{36}=6$

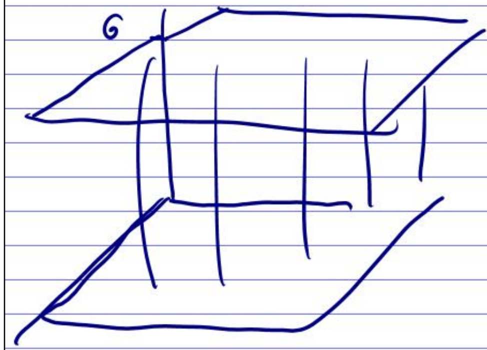
$\sqrt{2^2+16+4^2}=\sqrt{36}=6$

$\sqrt{36+4+0}=\sqrt{40}$



$y \geq 0$

Panel 3



$0 \le \boxed{z \le 6}$

Ball

$x^2+y^2+z^2 \le 3$

$x^2+y^2+z^2 > 2z$

$x^2+y^2+z^2-2z > 0$

$x^2+y^2+(z-1)^2 > 1$

outside sphere, centered at $(0,0,1)$ radius 1

Panel 4

$4x^2+4y^2+4z^2-8x+16y=1$

$4(x^2-2x)+4(y^2+4y)+4z^2=1$

$4(x-1)^2+4(y+2)^2+4z^2=21$

$(x-1)^2+(y+2)^2+z^2=\frac{21}{4}$

Panel 5

Calc 3 - Quiz #1

① Find the distance between $P(-1,2,0)$ and $Q(2,1,1)$.

② Find radius of sphere $x^2+y^2+z^2-6x+4y-2z=11$

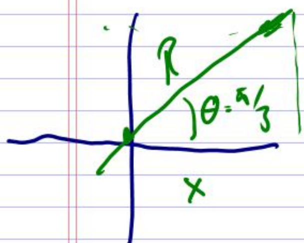
Panel 6

③ Describe 3D object given by $x^2+z^2=4$

④ Find a vector in direction $\langle -3,4,5\rangle$ with length 2.

Panel 7

Other ways to describe vectors: Find a vector of length 2 that makes an angle of $\pi/3$ with positive x-axis.



$$\frac{x}{R} = \cos(\theta)$$

$$\frac{y}{R} = \sin(\theta)$$

$$x = R\cos(\theta) = 2 \cdot \cos(\tfrac{\pi}{3}) = 2 \cdot \tfrac{1}{2} = \underline{1}$$

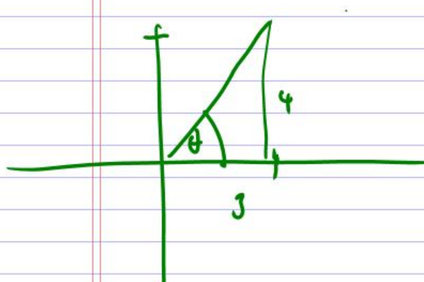
$$y = R\sin(\theta) = 2\sin(\pi/3) = 2 \cdot \tfrac{\sqrt{3}}{2} = \underline{\sqrt{3}}$$

Panel 8

Other way around: find angle that $\vec{v} = 3i + 4j$ makes with the positive x-axis.

$$v = 3i + 4j = \cancel{(3,4)} \langle 3,4 \rangle = 5 \cdot \langle \cos(53°), \sin(53°) \rangle$$

$$R = \sqrt{9+16} = \sqrt{25} = 5$$



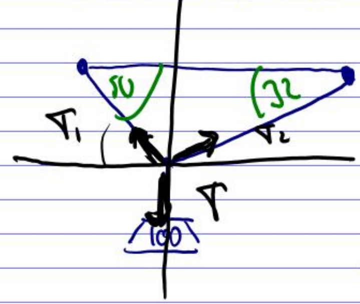
$$R\sin(\theta) = 4 \qquad \frac{\sin(\theta)}{\cos(\theta)} = \frac{4}{3}$$

$$R\cos(\theta) = 3$$

$$\arctan(\tfrac{4}{3}) = \underline{\underline{53°}}$$

Panel 9

What are vectors good for: a 100 lb weight hangs from two wires as shown. Find the forces T_1 and T_2 acting on the wires and their magnitudes.



$T_1 = \|T_1\| \langle -\cos(50), \sin(50) \rangle$

$T_2 = \|T_2\| \langle \cos(32), \sin(32) \rangle$

$T_1 + T_2 = T = \langle 0, 100 \rangle$

2 equations:

$-\|T_1\|\cos(50) + \|T_2\|\cos(32) = 0$

$\|T_1\|\sin(50) + \|T_2\|\sin(32) = 100$

solve $\|T_1\| = 85.64$, $\|T_2\| \approx 64.91$ $\quad T_1 = \sim$, $T_2 = --$

Panel 10

Knows vectors (i, j, k)

$\langle \quad \rangle$

$R \langle \cos, \sin \rangle$

add, subtract

Multiply? $\vec{v} + \vec{w} = \langle v_1, v_2 \rangle + \langle w_1, w_2 \rangle = \langle v_1 + w_1, v_2 + w_2 \rangle$

$\cancel{\vec{v} \cdot \vec{w} = \langle v_1 w_1, v_2 w_2 \rangle}$

Ex: $\cancel{\langle 2, 3 \rangle \cdot \langle 4, 5 \rangle = \langle 8, 15 \rangle}$

Caution: If $A \cdot B = 0 \Rightarrow$ either A or B ~~are~~ zero

Ex: $\langle 4, 0 \rangle \cdot \langle 0, 1 \rangle = \langle 0, 0 \rangle$ Bummer!

Panel 11

Dot Product:

$v = \langle v_1, v_2, v_3 \rangle$

$w = \langle w_1, w_2, w_3 \rangle$

Define $\vec{v} \cdot \vec{w} = v_1 \cdot w_1 + v_2 w_2 + v_3 w_3 \in \mathbb{R}$

dot

Panel 12

Examples of Dot product

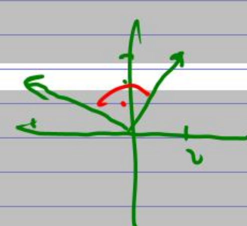
① $\langle 3, 5 \rangle \cdot \langle -1, 2 \rangle =$ $V_1 W_1 + V_2 W_2$

$(3)(-1) + (5)(2)$

$-3 + 10$

7

② $\langle 2, 3 \rangle \cdot \langle -3, 2 \rangle =$ $-6 + 6 = \boxed{0}$



③ $\langle 1, -3, 4 \rangle \cdot \langle 1, 5, 2 \rangle =$ $1 \cdot 1 + -3 \cdot 5 + 4 \cdot 2 = -6$

Panel 13

Properties of Dot Product

a) $\vec{a} \cdot \vec{a} = \|a\|^2$

b) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

c) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

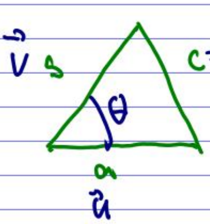
$\vec{a} = \langle a_1, a_2, a_3 \rangle : \quad \vec{a} \cdot \vec{a} = \langle a_1, a_2, a_3 \rangle \cdot \langle a_1, a_2, a_3 \rangle =$

$= a_1^2 + a_2^2 + a_3^2 = \|\vec{a}\|^2$

Panel 14

Theorem: If u and v are non-zero vectors in $\mathbb{R}^2$ then

$\dfrac{u \cdot v}{\|u\| \cdot \|v\|} = \cos(\theta)$, or dot-product gives you angle between vectors



$c = v - u$ law of cosines: $c^2 = a^2 + b^2 - 2ab\cos(\theta)$

$\|v - u\|^2 = \cancel{\|v\|^2} + \cancel{\|u\|^2} - \cancel{2}\|v\|\|u\|\cos(\theta)$

$(v-u)\cdot(v-u) =$ — " —

$v \cdot v - v \cdot u - u \cdot v + u \cdot u =$ — " —

$\cancel{\|v\|^2} - \cancel{2}\, u \cdot v + \cancel{\|u\|^2} =$ — " —

$\cos(\theta) = \dfrac{u \cdot v}{\|u\| \cdot \|v\|}$