Panel 1

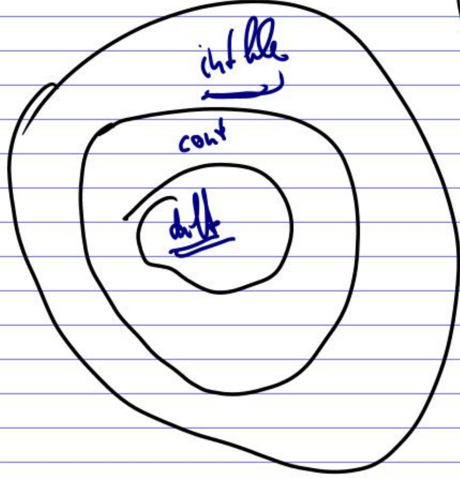
Last time

Review of Calc 1 + 2 — limits

continuity

differentiability

integration

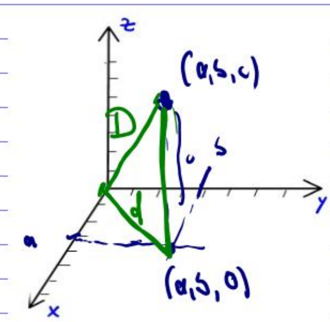


Panel 2

Distance in $\mathbb{R}^3$

$P(a, b, c)$

Distance between origin and P



$d = \sqrt{a^2 + b^2}$

$D = \sqrt{d^2 + c^2} = \sqrt{a^2 + b^2 + c^2}$

Distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Panel 3

$P(3,2,1)$ and $Q(4,5,6)$

Find distance to origin and distance P to Q

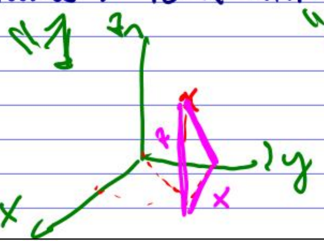
$$D = \sqrt{(3-4)^2+(2-5)^2+(1-6)^2} = \sqrt{1+9+25} = \underline{\underline{\sqrt{35}}}$$

Dist. of P to origin: $D = \sqrt{9+4+1} = \sqrt{15}$

Dist. of P to xy-plane? To x-axis?

"1"   "$\sqrt{5}$"

Dist. to y-axis. $\sqrt{10}$



Panel 4

<u>3D Objects</u>

$P(x,y,z) \Rightarrow d = \sqrt{x^2+y^2+z^2} \Rightarrow d^2 = x^2+y^2+z^2$

All (x,y,z) that are a fixed distance to the origin form a <u>sphere</u>: $x^2+y^2+z^2 = 9$ (radius 3)

<u>Def</u>: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = r^2$ is sphere, radius r, centered at (x_0,y_0,z_0)

<u>Ex</u>: $x^2+y^2+z^2-2x-4y+8z+17=0$

$$x^2-2x+1 \quad +y^2-4y+4 \quad +z^2+8z+16 = -17+1+4+16$$

$$(x-1)^2+(y-2)^2+(z+4)^2 = 4 \quad \text{center } (1,2,-4),\ r=2$$

Panel 5

Def: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$ is disk centered at $P(x_0,y_0,z_0)$ with radius r.

Ex: Find the center + radius of the sphere
$x^2+y^2+z^2+10x+4y+2z-19=0$

$x^2+10x+25+y^2+4y+4+z^2+2z+1=49$

$(x+5)^2+(y+2)^2+(z+1)^2=49$

$C=(-5,-2,-1)$

$R=7$

Panel 6

Drawing 3D objects with Maple

Maple can easily draw 3D objects

```
> with(plots);
> implicitplot3d(z=y^2, x=-3..3, y=-3..3, z=-1..9);
> plot3d(x^2, x=-3..3, y=-3..3);
> implicitplot3d(z^2+y^2=4, x=-3..3, y=-3..3, z=-3..3);
```

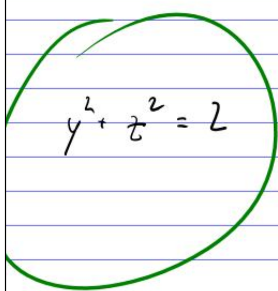
plot3d: $z=x^2+y^2$

implicitplot3d: $x^2+y^2+z^2=9$

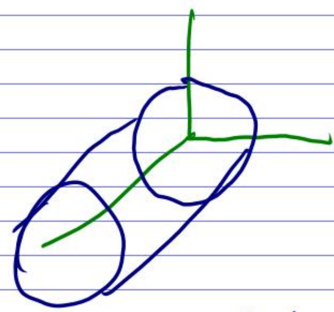
Panel 7

Ex: Use Maple to graph the following:

$x^2+y^2+z^2=4$



$y^2+z^2=2$



$z=\sin(x)\cdot\cos(y)$

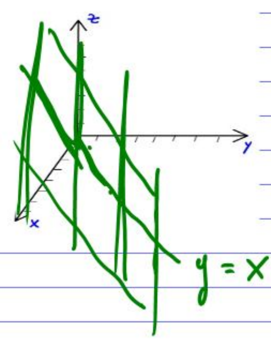
cylinder around z-axis, radius 3

$x^2+y^2=9$

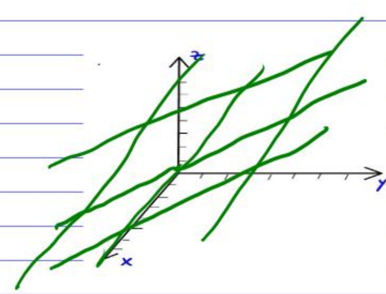
Panel 8

3D Objects

a) $y=x$



b) $z=\frac{1}{2}y$

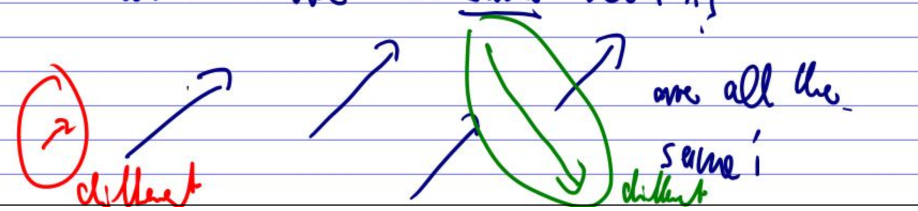


Panel 9

We understand points in 3D (and 2D).
Want to investigate more general objects: vectors

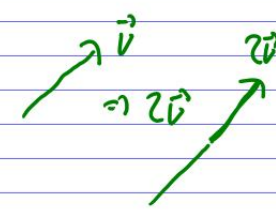
Def: A vector is a directed line segment, i.e. a part of a line with a given length and a fixed direction.
A vector from point A to B is written as
$\vec{v} = \vec{AB}$.

Note: All line segments of same length and same direction are the same vector !!



Panel 10

Vector Math, geometrically:

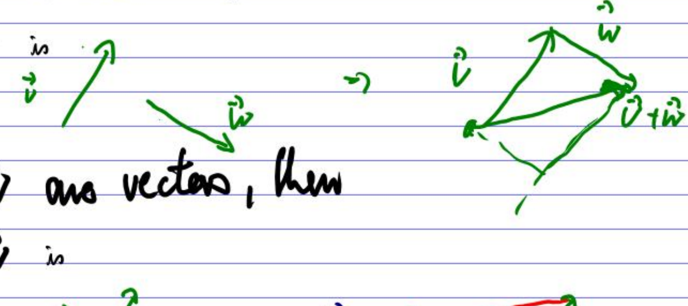


If $\vec{v}$ is a vector, then
$k \cdot \vec{v}$ is ~~[illegible]~~

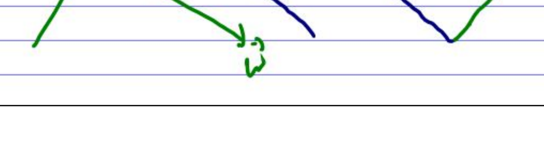
$k \in \mathbb{R}$ k-times v

If $\vec{v}$ is a vector then

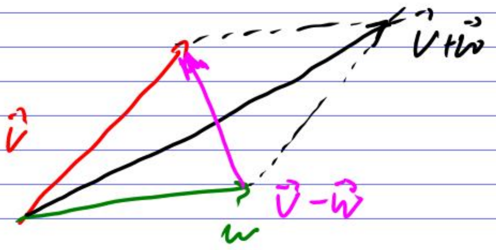
If $\vec{v}, \vec{w}$ are vectors, then
$\vec{v} + \vec{w}$ is



If $\vec{v}, \vec{w}$ are vectors, then
$\vec{v} - \vec{w}$ is



Panel 11



Panel 12

Vector Math, Algebraically vs. Geometrically

Algebraically v is described by components:

$\vec{v} = \langle v_1, v_2 \rangle$ or $\vec{v} = \langle v_1, v_2, v_3 \rangle$

?

HW

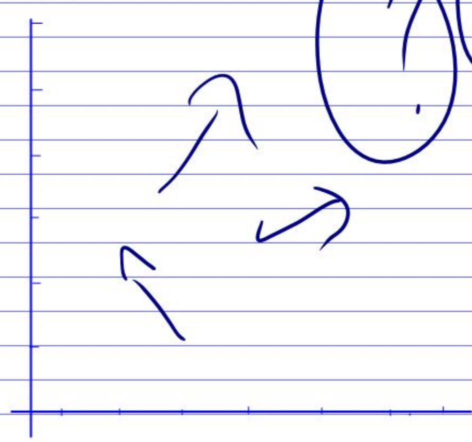
Ex: Suppose $\vec{v} = \langle 1,2 \rangle$, $\vec{w} = \langle 3,1 \rangle$. Find

$\vec{v} + \vec{w}$ $\quad \langle 1,2 \rangle + \langle 3,1 \rangle =$

$\langle 4,3 \rangle$

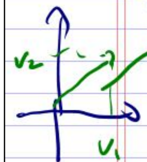
$\vec{v} + 2\vec{w}$

$3\vec{v} - \vec{w}$



Panel 13

Vectors: Some Definitions



Def: The length or norm of a vector $\vec{v} = \langle v_1, v_2 \rangle$ is:
$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}.$$

Def: A unit vector $\vec{v}$ is a vector such that
$$\|\vec{v}\| = 1$$

Note: If $\vec{v} = \langle v_1, v_2 \rangle$ is any non-zero vector, then
$$\vec{u} = \frac{1}{\|\vec{v}\|} \vec{v}$$
is a unit vector pointing in the same direction as $\vec{v}$.

Panel 14

Ex: a) $\langle \frac{1}{2}, \frac{3}{4} \rangle$ who is a unit vector?

b) $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

Ex: Find unit vector in the direction of $\vec{v} = \langle 1, -5 \rangle$ and $\vec{w} = \langle 3, 2, -1 \rangle$

unit vector, direction v : $\frac{1}{\sqrt{26}} \langle 1, -5 \rangle =$

$$\left\| \left\langle \frac{1}{\sqrt{26}}, -\frac{5}{\sqrt{26}} \right\rangle \right\| = \sqrt{\frac{1}{26} + \frac{25}{26}} = 1$$

$\vec{w}$: $\frac{1}{\sqrt{14}} \langle 3, 2, -1 \rangle$

Panel 15

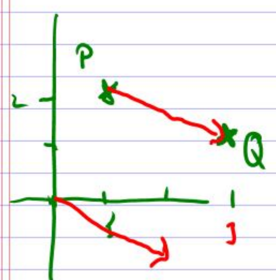
scalar

Ex.: If $\vec{V} = -2\vec{i} + 3\vec{k}$ write $\vec{v}$ in standard notation and find $\|\vec{v}\| = \sqrt{13}$ use

$v = -2\langle 1,0,0\rangle + 3\langle 0,0,1\rangle = \underline{\langle -2, 0, 3\rangle}$

$i = \langle 1,0,0\rangle$, $j = \langle 0,1,0\rangle$, $k = \langle 0,0,1\rangle$ are basic unit vectors
"x" "y" "z"

Note: If $P(1,2)$ and $Q(3,1)$ find $\vec{v} = \vec{PQ}$: $= \langle 2,-1\rangle$



$\vec{v} = \vec{PQ} = \vec{Q} - \vec{P} =$

$= (3,1) - \langle 1,2\rangle = \underline{\langle 2,-1\rangle}$

$\boxed{\text{Quiz on Friday}}$