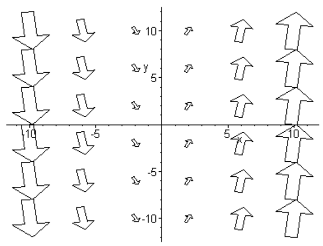
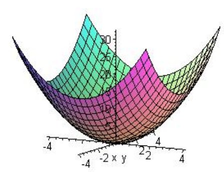
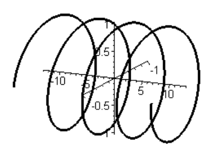
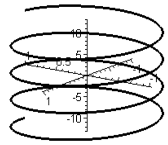
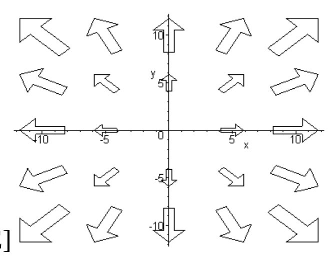
Definitions and Concepts:

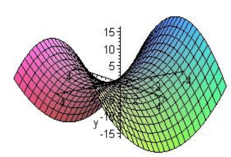
- Vector, Angle between two vectors, Unit vector, dot product, cross product, projections
- Tangent vector to a curve, normal vector to a curve, binormal vector
- Velocity, speed, and acceleration, tangential and normal component of acceleration, curvature
- Lines, planes, and distances
- Limit of a function $z = f(x, y)$
- Continuity of a function $z = f(x, y)$
- partial derivative of a function f(x,y); directional derivative
- gradient and its properties, curl and divergence
- the procedure to find relative extrema of a function f(x, y)
- double and triple integrals, including polar coordinates
- What does it mean when a "line integral of a vector field F is independent of the path"?
- What is Green's Theorem?
- What is Gauss' Theorem?
- What is Stoke's Theorem?
- For what type of curve can you apply Green's theorem?
- For what type of surface can you apply the Divergence theorem?
- For what type of curve can you apply Stoke's theorem?

Picture problem: Match the following pictures with the algebraic expressions below.

[A] 

[B] 
[C] 

[D] 
[E] 

[F] 

(1) $f(x, y) = x^2 + y^2$ (2) $f(x, y) = x^2 - y^2$ (3) $r(t) = \langle \cos(t), \sin(t), t \rangle$

(4) $r(t) = \langle \cos(t), t, \sin(t) \rangle$ (5) $F(x, y) = \langle x, y \rangle$ (6) $F(x, y) = \langle 1, x \rangle$

Vectors: Suppose $u =< 7,-2,3 >$, $v =< -1,4,5 >$, and $w =< -2,1,-3 >$

- Are *u* and *v* orthogonal, parallel, or neither?
- Find the (cos of the) angle between *v* and *w*
- Find $u \cdot v$ (dot product), $u \times v$ (cross product), $u \cdot (v \times w)$, and $\|u\|$
- Find the projection of *w* onto *u* and the projection of *u* onto *w*

Lines and Planes

- Find the equation of the plane spanned by $< 1,3,-2 >$ and $< 2,1,2 >$ through the point $P(1,2,3)$

- Find the equation of the plane through $P(1,2,3)$, $Q(1,-1,1)$, and $R(3,2,1)$
- Find the equation of the plane parallel to $x - y + z = 2$ through $P(0,2,0)$
- Find the equation of the line through $P(1,2,3)$ and $Q(1,-1,1)$
- Some distance questions

Vector valued functions:

- If $r(t) = < 4t, t^2, t^3 >$, find $r'(t)$, $r''(t)$, $\frac{d}{dt}\|r(t)\|$
- If $r(t) = < e^t, 3t^3, \frac{3}{6t} >$ some curve, find $\int_1^2 r(t)dt$
- If $r(t) = < t, \frac{1}{t} >$, find *T(t), N(t)*, a_t and a_n
- Repeat the previous exercise for $r(t) = < e^t \cos(t), e^t \sin(t) >$ for $t = \frac{\pi}{2}$

Motion in space:

- If $r(t) = < t, 3t^2, \frac{t^2}{2} >$ represents the position vector of a particle, find the velocity, speed, and acceleration, as well as tangential and normal components of the acceleration
- A baseball is hit 3 feet above ground at 100 feet per second and at an angle of Pi/4 with respect to the ground. Find the maximum height reached by the baseball. Will it clear a 10-foot high fence located 300 feet from home base?

Limits and Continuity: Determine the following limits as *(x,y) -> (0,0)*, if they exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy+1}{x^2+y^2+1} \qquad \lim_{(x,y)\to(0,0)} \frac{xy+1}{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

Gradient and Friends. Find the indicated expressions:

(a) If $f(x,y,z) = ye^x + x\ln(y) + z^3$, find ∇f (i.e. the gradient of *f*)

(b) If $F(x,y,z) = \left\langle x - x^2y, y^2x, x^5y - z \right\rangle$, find *div(f)* (i.e. the divergence of *f*)

(c) If $F(x,y,z) = \left\langle (2z^2x), (x^2y), (z^2 + x^2) \right\rangle$, find *curl(f)*

Differentiation: Find the indicated derivatives for the given function:

- Suppose $f(x,y) = 2x^3y^2 + 2y + 4x$, find f_x, f_y, f_{xx}, f_{xy}, f_{yy}, f_{yx}, f_{xyy}, f_{yxy}, and f_{yyx}
- Suppose $f(x,y) = x^2e^y$. Find the maximum value of the directional derivative at (-2, 0) and compute a unit vector in that direction.
- Use the <u>definition</u> of f_x to find it

Max/Min Problems: Compute the relative extrema for $f(x,y) = 3x^2 - 2xy + y^2 - 8y$ and $f(x,y) = 4xy - x^4 - y^4$.

Conservative vector fields: TBA

Integration: Find the following integrals. As always, you may use Maple to help you out.

- $\int_0^2 \int_{x^2}^{x} (x^2 + 2y)\,dy\,dx$
- $\int_{-3}^{3} \int_0^{\sqrt{9-x^2}} \sqrt{x^2 + y^2}\,dy\,dx$
- evaluate $\iint_R \frac{y}{x^2 + y^2}$ where R is a triangle bounded by $y = x$, $y = 2x$, $x = 2$
- $\iint_R e^{x^2}\,dA$ where R is the triangular region bounded by y = 0, y = x, and x = 1
- $\iiint_E z\,dV$, where E is the solid tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$
- $\int_C x + y^2 ds$ where C is a line segment given by $r(t) = <3t, 4t>$, $0 \le t \le 1$
- $\int_C F \cdot dr$ where $F(x, y) = <-y, x>$ and C is the curve given by $r(t) = <2\cos(t), 2\sin(t)>$, $0 \le t \le \pi$
- Find the work done by a force field $F(x, y, z) = \langle -x, -y, 2 \rangle$ on a particle as it moves along the helix C given by $r(t) = <\cos(t), \sin(t), t>$, $0 \le t \le 3\pi$
- $\int_C y^2 dx + x dy$ where C is the curve $r(t) = <(t^2 - 1), t>$, $-1 \le t \le 1$
- $\int_C F \cdot dr$ where $F(x, y) = <2xy^3 + y\sin(x), 3x^2y^2 - \cos(x)>$ and C is the boundary of the square with corner point (0,0), (1,0), (1,1), and (0, 1), oriented counter-clockwise.
- $\int_C F \cdot dr$ where $F(x, y) = <2xy^3 - 2xy + 1, 3x^2y^2 - x^2>$ and C is the lower half of the unit circle, from (-1,0) to (1,0).
- $\int_C (3x^2 y - y^3)dx + x^3 dy$ where C is the boundary of the square with corner point (0,0), (1,0), (1,1), and (0, 1), oriented counter-clockwise.
- Find the surface integral $\iint_S x - 2y + z dS$, where S is the surface $z = 10 - 2x + 2y$ such that x is between 0 and 2 and y is between 0 and 4.
- Evaluate the flux integral $\iint_S \vec{F} \cdot \vec{n}\, dS$ where $F(x, y, z) = <x, y, z>$ and S is $x^2 + y^2 + z^2 = 4$
- Evaluate $\int_C \vec{F}\, dr$ where $F(x, y, z) = <z^2, x^2, y^2>$ and C is the boundary of the surface S given by $z = 4 - x^2 - y^2$ and $z \ge 0$, oriented counter-clockwise.

- Evaluate $\int_C \vec{F}\, d\vec{r}$ where $F(x, y, z) =< -y^2, x, z^2 >$ and C is the curve bounding the ellipse S consisting of the intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$

- Evaluate $\iint_S curl(F)dS$ where $F(x, y, z) =< xz, yz, xy >$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ above the *xy*-plane.