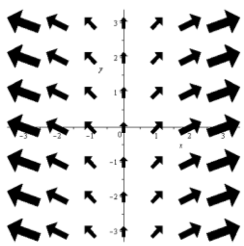
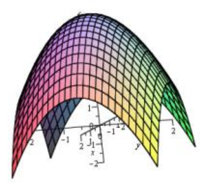
Calc 3 – Final Exam

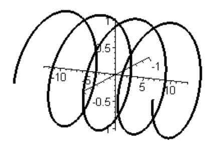
This is a take-home, open-book, open notes exam; you even may use Maple or Wolfram Alpha to assist you in calculations. You must, however, complete it *entirely on your own*. It is due on Friday, Dec 13, 2013 – no exception. Please indicate clearly where each problem starts and do not forget to put your name on your exam.

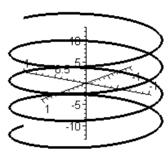
1. Please state the following:
   a) an equation relating the dot product of two vectors with the angle between them
   b) the definition of the gradient of a function $f(x, y, z)$ and its properties
   c) the main difference between Green's and Stoke's Theorem as far as the vector field $\vec{F}$ is concerned
   d) the Divergence Theorem (also known as Gauss' Theorem)

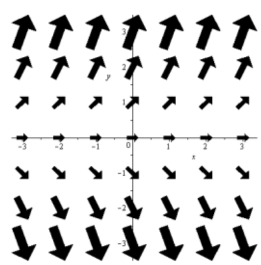
2. Match the following pictures with the algebraic expressions below.

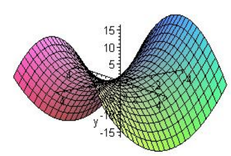
[A] 

[B] 

[C] 

[D] 

[E] 

[F] 

(1) $f(x, y) = 6 - x^2 - y^2$ (2) $f(x, y) = y^2 - x^2$ (3) $r(t) = \langle \cos(t), \sin(t), t \rangle$

(4) $r(t) = \langle \cos(t), t, \sin(t) \rangle$ (5) $F(x, y) = \langle x, 1 \rangle$ (6) $F(x, y) = \langle 1, y \rangle$

3. Determine if the plane through the points $P(1,0,0)$, $Q(0,2,0)$, and $R(0,0,3)$ is perpendicular to the plane given by the equation $2x - 2y - 3z = 1$

4. A baseball is hit 4 feet above ground at an initial velocity of $< 80, 80 >$ feet per second. Find the maximum height reached by the baseball. Will it clear a 15-foot high fence located 350 feet from home base?

5. Determine the following limits, if possible, or explain why they don't exist.

$\lim_{(x,y)\to(0,0)} \frac{x+2y+3}{x^2+2y^2+3}$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

6. Find all critical points and test them for relative extrema for the function $f(x,y) = -x^3 + 3xy - \frac{3}{2}y^2$ (*Hint: There are two critical points*)

7. Evaluate the following integrals:

   a) $\iint_R \cos(x^2)\, dA$, where R is the triangular region bounded by $y = 0$, $y = x$, and $x = 1$

   b) $\int_C x^2 + y^2\, ds$, where C is the curve given by $r(t) =< 2\cos(t), 2\sin(t) >$ for $0 \le t \le \pi$

   c) $\int_C \vec{F}\, d\vec{r}$, where $\vec{F}(x,y,z) =< -y^2, x, z >$ and C is the line segment from $P(-1,-1,0)$ to $Q(1,2,3)$

   d) The flux of the vector field $\vec{F}(x,y,z) =< x, y, z >$, where S is the portion of the surface $z = 10 - 2x - 2y$ between the coordinate planes.

8. For the following integrals there could be more than one way to evaluate them. Use the most convenient method and quote the appropriate theorem, if necessary.

   a) $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y,z) =< 2xz^2, z\cos(y), 2x^2z + \sin(y) >$ and C a curve from (1,0,1) to (1,4,4)

   b) $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) =< 2xy^3 + x\cos(x), 3x^2y^2 - \sin(y)e^y >$ and C is the closed curve given by the boundary of the square with corner points (-1,-1), (-1,1), (1, -1), and (1,1).

   c) $\int_C (3x^2y - y^2)dx + x^3dy$ where C is the closed curve given by the boundary of the triangle with corner points (0,0), (0,1), and (1, 0), oriented counter-clockwise.

   d) $\iint_S \vec{F} \cdot \vec{n}\, dS$ where $F(x,y,z) =< x + z^2, 2y - \cos(xz), 3z >$ and S is given by $x^2 + y^2 + z^2 = 9$

   e) $\int_C \vec{F}\, dr$ where $F(x,y,z) =< z^2, x^2, y^2 >$ and C is the boundary of the surface S given by $z = 1 - x - y$, restricted by the coordinate planes and oriented counter-clockwise.