7a. **Chain Rule:** Find the following derivatives of the (sometimes implicitly defined) functions:

a) If $z = x^3 y + 3xy^4$ and $x = \sin(2t)$ and $y = \cos(t)$, find $\frac{\partial z}{\partial t}$ at *t=0*

at $t=0$ we have $x(0)=0$, $y(0)=1$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (3x^2 y + 3y^4)\frac{\partial x}{\partial t} + (x^3 + 12xy^3)\frac{\partial y}{\partial t}$$

$\frac{\partial x}{\partial t} = 2\cos(2t)$ and $\frac{\partial y}{\partial t} = -\sin(t)$

Thus, at $t=0$, $\left.\frac{\partial z}{\partial t}\right|_{t=0} = (3\cdot 0\cdot 1 + 3\cdot 1^4)\,2 + (0+0)\cdot 0 = \boxed{6}$

$\Rightarrow \left.\frac{\partial x}{\partial t}\right|_{t=0} = 2$, $\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0$

b) If $x^3 + y^3 = 6xy$ defines *y* as a function of *x* implicitly, find $\frac{\partial y}{\partial x}$

$$\frac{\partial}{\partial x}(x^3+y^3) = \frac{\partial}{\partial x}(6xy) \iff 3x^2 + 3y^2\frac{\partial y}{\partial x} = 6y + 6x\frac{\partial y}{\partial x}$$

$$\Rightarrow \quad 3x^2 - 6y = 6x\frac{\partial y}{\partial x} - 3y^2\frac{\partial y}{\partial x}$$

$$\Rightarrow \quad \frac{3x^2 - 6y}{6x - 3y^2} = \frac{\partial y}{\partial x}$$

c) Let $z = e^x \cos(y)$ and $x = s \cdot t$ and $y = \sqrt{s^2 + t^2}$, find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$

Skip for now

d) Find the rate of change of *y* with respect to *t* of $x^2 + y^2 + z^2 = 1$ for t = 1, assuming that *x, y,* and *z* are all functions of *t* and that *x(1) = 1, y(1) = 2, z(1) = 3,* and *x'(1) = 0* and *z'(1)* = ½

$$2xx' + 2yy' + 2zz' = 0 \Rightarrow ?$$

$$2\cdot 1\cdot 0 + 2\cdot 2\cdot\frac{\partial y}{\partial t} + \cancel{2}\cdot 3\cdot\frac{1}{\cancel{2}} = 0$$

$$\frac{\partial y}{\partial t} = -\frac{3}{4}$$