① Find $\vec{a} + \vec{b}$, $2\vec{a} + 3\vec{b}$, $\|\vec{a}\|$, and $\|\vec{a} - \vec{b}\|$ for

a) $a = 4i + j$, $b = i - 2j$

Review

b) $a = i + 2j - 3k$, $b = -2i - j + 5k$

c) $a = 2i - 4j$, $b = 2j - k$

② Find a unit vector that has the same direction as:

a) $-3i + 7j$

Review

b) $8i - j + 4k$

③* If $\vec{v}$ lies in the 1st quadrant of R^2 and makes an angle of $\pi/4$ with the x-axis and $\|v\| = 4$, find $\vec{v}$ in component form, i.e. $\vec{v} = \langle a, b \rangle$, where a, b are the components to find.

④ Find the unit vectors parallel to the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.

⑤ If A, B, C are the vertices of a triangle, find $\vec{AB} + \vec{BC} + \vec{CA}$

⑥ If $\vec{r} = \langle x, y, z \rangle$ and $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$, describe the set of all points (x, y, z) such that

$$\| \vec{r} - \vec{r}_0 \| = 1$$

⑦ Which expression makes sense, where $\cdot$ is dot-product

a) $\left(\vec{a} \cdot \vec{b} \right) \vec{c}$ b) $\|\vec{a}\| \left(\vec{b} \cdot \vec{c} \right)$ c) $\vec{a} \cdot \left(\vec{b} + \vec{c} \right)$

d) $\vec{a} \cdot \vec{b} + \vec{c}$ e) $\|\vec{a}\| \cdot \left(\vec{b} + \vec{c} \right)$

⑧ Find dot product of two vectors if their lengths are 6 and $\frac{1}{3}$ and the angle between them is $\frac{\pi}{4}$.

⑨ Find $\vec{a} \cdot \vec{b}$ for

a) $\vec{a} = \langle -2, \frac{1}{3} \rangle$, $\vec{b} = \langle -5, 12 \rangle$

b) $\vec{a} = \langle 4, 1, \frac{1}{4} \rangle$, $\vec{b} = \langle 6, -3, -8 \rangle$

⑩ Find angle between $\vec{a}$ and $\vec{b}$ for:

a) $\vec{a} = \langle \sqrt{3}, 1 \rangle$, $\vec{b} = \langle 0, 5 \rangle$

b) $\vec{a} = \langle 4, 0, 2 \rangle$, $\vec{b} = \langle 2, -1, 0 \rangle$

c) $\vec{a} = i + 2j - 2k$, $\vec{b} = 4i - 3k$

⑪ Are the vectors orthogonal, parallel, or neither?

a) $\vec{a} = \langle -5, 3, 7 \rangle$, $\vec{b} = \langle 6, -8, 2 \rangle$

b) $\vec{a} = \langle -3, 9, 6 \rangle$, $\vec{b} = \langle 4, -12, -8 \rangle$

c) $\vec{a} = \langle 7, 0, -2 \rangle$, $\vec{b} = \langle 2, 5, 7 \rangle$

(12) For what values of b are the vectors $\langle -6, b, 2 \rangle$ and $\langle b, b^2, b \rangle$ orthogonal?

(13) Find the directional angles (cosines) of $\langle 3, 4, 5 \rangle$

(14) * If $\vec{r} = \langle x, y, z \rangle$, $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, show that the equation

$$(\vec{r} - \vec{a}) \cdot (\vec{r} - \vec{b}) = 0$$

is a sphere. Find its center and radius.

(15) * Show that if $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal, then $\vec{u}$ and $\vec{v}$ must have the same length.