Panel 1

Last time:

cancels $\sqrt{f_x^2+f_y^2+1}$

Flux of a 3D vector field $\vec{F} = \langle M, N, P \rangle$

$$\iint_S \vec{F}\cdot\vec{n}\, dS = \iint_R \langle M,N,P\rangle \langle -f_x, -f_y, 1\rangle\, dA$$

$$= \iint_R -f_x M - f_y N + P\, dA$$

$z = f(x,y)$ gives $S$ over $R$

Gauss thm: If $S$ is a closed surface then

$$\oiint_S \vec{F}\cdot\vec{n}\, dS = \iiint_Q \operatorname{div}(\vec{F})\, dV$$

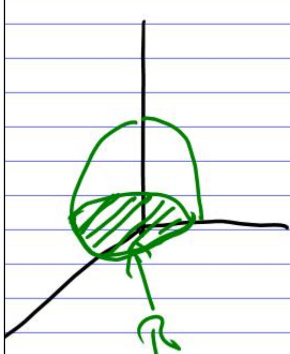
Panel 2

Ex: Let $S$ be the portion of the surface $z = 1 - x^2 - y^2$ above the $xy$-plane. Find the flux of the vector field $\vec{F} = \langle x, y, z\rangle$ across $S$.

$$\iint_S \vec{F}\cdot\hat{n}\, dS = \iint_R -f_x M - f_y N + P\, dA =$$

$$= \iint_R 2x^2 + 2y^2 + (1 - x^2 - y^2)\, dA =$$

$$= \iint_{R\,(\text{circle})} 1 + x^2 + y^2\, dA = \int_0^{2\pi}\int_0^1 (1 + r^2)\, r\, dr\, d\theta = \underline{\underline{W.A.}}$$

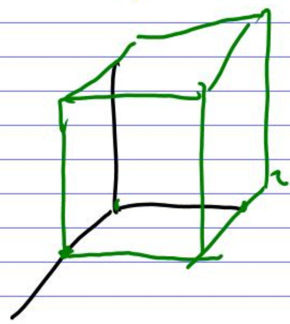


Panel 3

Div. Thm. uses Tripple Integrals:

Ex: $\iiint_Q (xyz)\, dV$ , $Q = [0,1] \times [0,2] \times [0,3]$

$w = xyz$



$$\int_0^3 \int_0^2 \int_0^1 xyz \, dx\, dy\, dz =$$

$$\int_0^1 \int_0^3 \int_0^2 xyz \, dy\, dz\, dx =$$

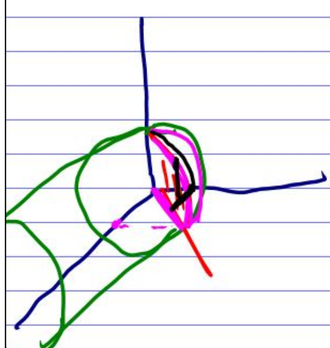
$$\int_0^3 \int_0^1 \int_0^2 xyz \, dy\, dx\, dz =$$

$\vdots$

Panel 4

Ex: Let Q be the region in the first octant cut from the cylinder $y^2+z^2=1$ by $y=x$ and $x=0$. Find

$$\iiint_Q z \, dV = \int_0^1 \int_0^y \int_0^{\sqrt{1-y^2}} z \, dz\, dx\, dy = \underline{\underline{W.A.}}$$



depend on x, y

depend on y

\#

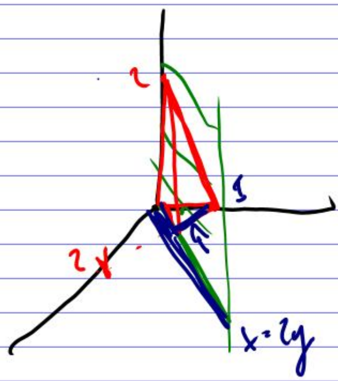
Panel 5

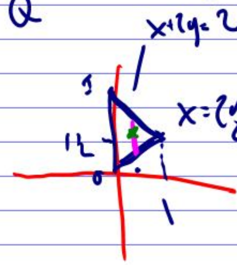
Volumes $\iint_R dA = \text{area}(R)$ , $\iiint_Q dV = \text{vol}(Q)$

Find volume of tetrahedron T bounded by $x+2y+z=2$, $x=2y$, $x=0$, and $z=0$

$$\iiint_Q dV = \int_0^1 \int_{x/2}^{1-\frac{1}{2}x} \int_0^{2-x-2y} dz\, dy\, dx =$$

W.A.

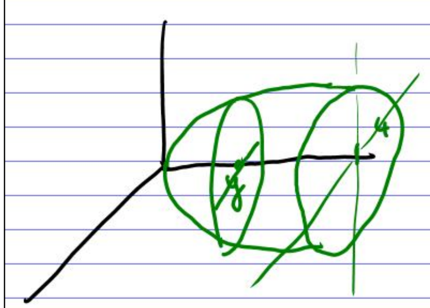



dzdxdy = 2 integrals

Panel 6

Ex: $\iiint_E \sqrt{x^2+z^2}\, dV$ , E bounded by $y=x^2+z^2$ and $y=4$




$$\int_0^4 \underbrace{\iint}_{\substack{\text{disk of} \\ \text{radius } \sqrt{y}}} \sqrt{x^2+z^2}\, \underbrace{d?\, d?}\, dy =$$

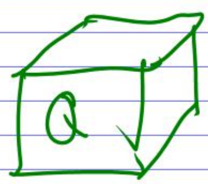
$$= \int_0^4 \int_0^{2\pi} \int_0^{\sqrt{y}} \sqrt{r^2}\, r\, dr\, d\theta\, dy = \text{W.A.}\left(\frac{128}{15}\pi\right)$$

Panel 7

Now moving on to Flux Integrals and Div Thm.

Find flux of the vector field $F(x,y,z) = \langle 2x, 3y, z^2 \rangle$ across the unit cube.

$$\iint_S \vec{F}\cdot\hat{n}\, dS = 6 \text{ integrals}$$



$$\iiint_Q \operatorname{div}(\vec{F})\, dV = \int_0^1\int_0^1\int_0^1 2+3+2z \; dz\,dy\,dx \checkmark$$

Panel 8

Ex. Find flux of $\vec{F}(x,y,z) = \langle y, x, z \rangle$ across

$$x^2+y^2+z^2 = R^2 \qquad z = \pm\sqrt{R^2-x^2-y^2} = f(x,y)$$

$$f_x = 1$$

$$\iint_S \vec{F}\cdot\hat{n}\, dS =$$

$$\iiint_{\substack{\text{sphere}\\ \text{radius } R}} 1\, dV = \text{vol (sphere)} = \underline{\frac{4}{3}\pi R^3}$$

Panel 9

Ex: $Q$ region bdd by $z=4-x^2$, $y+z=5$, $xy$ and $xz$-planes.

Let $\vec{F}=\langle x^3+\sin(z), x^2y+\cos(z), e^{x^2+y^2}\rangle$, find $\iint_R \vec{F}\cdot\vec{n}\,dS$

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint 3x^2+x^2\,dV =$$

$$= \int\int\int 4x^2\,d\ \,d\ \,d$$

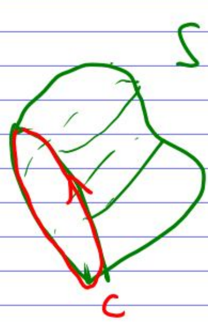
HW

Panel 10

Last: Stoke's Theorem — 3D

Let $S$ be an orientable surface with normal vector $\vec{n}$. Suppose $S$ is bounded by a simple closed curve $C$. If $\vec{F}$ is a vector field with cont. partials then

$$\oint_C \vec{F}\,d\vec{r} = \iint_S \mathrm{curl}(\vec{F})\cdot\vec{n}\,dS$$



Panel 11

⇒ Green's Theorem

2D

$$\oint_C \vec{F}\, d\vec{r} = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$$

closed curve C
R is inside

or

$$\oint \vec{F}\, d\vec{r} = \iint_R \mathrm{curl}(\vec{F}) \cdot \vec{k}\, dA$$

Gauss Theorem:

3D

$$\oiint_S \vec{F} \cdot \vec{n}\, dS = \iiint_Q \mathrm{div}(\vec{F})\, dV$$

closed surface S
Q is inside

Stoke's Theorem

$$\oint_C \vec{F}\, d\vec{r} = \iint_S \mathrm{curl}(\vec{F}) \cdot \vec{n}\, dS$$

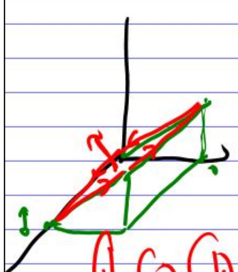
C is the boundary of S
S never closed

Panel 12

Ex: Find the work done by $\vec{F} = \langle x^2, 4xy^3, y^2x \rangle$ on a particle that traverses the boundary of $z = y$ $= f(x,y)$ above $[0,1] \times [0,3]$

$\vec{n} \langle -f_x, -f_y, 1 \rangle$

$$\oint_C \vec{F}\, d\vec{r} = \int_C M\,dx + N\,dy + P\,dz = \text{4 integrals}$$



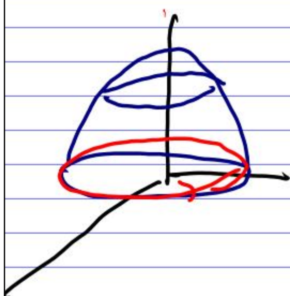
$$\overset{\text{Stokes}}{=} \iint_S \mathrm{curl}(\vec{F}) \cdot \vec{n}\, dS =$$

$$= \iint \langle 2xy, -y^2, 4y^3 \rangle \cdot \langle 0, -1, 1 \rangle\, dA$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ x^2 & 4xy^3 & y^2x \end{vmatrix} = (2xy, -y^2, 4y^3) = \int_0^1 \int_0^3 y^2 + 4y^3\, dy\, dx = \text{w.d.} =$$

Panel 13

Example. $F = \langle 2z, 3x, 5y \rangle$ a vector field,
$S$ paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$ and
$C$ the boundary of $S$ in $xy$-plane. Find
$\int \vec{F}\, d\vec{r}$ a) old-fashioned way
b) Stoke's thm



(a) $\oint_C \vec{F}\, d\vec{r} = \int_C M\,dx + N\,dy + P\,dz =$

$C: r(t) = \langle \underbrace{2\cos(t)}_{x}, \underbrace{2\sin(t)}_{y}, \underbrace{0}_{z} \rangle$

$= \int_0^{2\pi} 0\, dt + 3\cdot(2^2\cos^2(t))\, dt + 5y \cdot 0\, dt = 12 \int_0^{2\pi} \cos^2(t)\, dt$

Panel 14

Example. $F = \langle 2z, 3x, 5y \rangle$ a vector field,
$S$ paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$ and
$C$ the boundary of $S$ in $xy$-plane. Find
$\int \vec{F}\, d\vec{r}$ a) old-fashioned way
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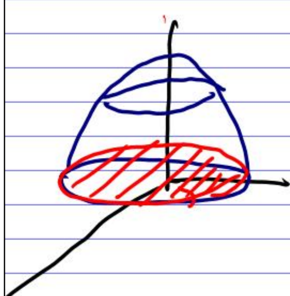

$\oint_C \vec{F}\, d\vec{r} = \iint \text{curl}(F) \cdot \overbrace{\langle -2x, -2y, 1 \rangle}^{\vec{n}}\, dA$

$= \iint \langle 5, 2, 3 \rangle \langle +2x, +2y, 1 \rangle\, dA =$

$= \iint_{\text{circle}} 10x + 4y + 3\, dA = \int_0^{2\pi}\int_0^{2} \left(10(2\cos(t)) + 4(2\sin(t)) + 3\right) r\, dr\, d\theta$

Panel 15

One more: use old-fashioned or Stoke?

C is triangle bdd by coord. plane and $2x+2y+z=6$. Let

$\vec{F} = (-y^2, z, x)$ and find $\int_C \vec{F} d\vec{r}$.

HW