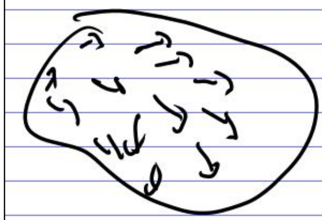
Panel 1

Vector field: a map from $\mathbb{R}^2 \to \mathbb{R}^2$ that assigns to every point in $D \subset \mathbb{R}^2$ a 2D-vector



Conservative Vector Field $\vec{F} = \langle M, N \rangle$: there is a function $f: \mathbb{R}^2 \to \mathbb{R}$ s.t.

$\mathrm{grad}(f) = \nabla f = \vec{F}$ i.e.

$f_x = M, \; f_y = N$

f is called potential function

Volume (D) : $\iiint_D dV$

$D \subset \mathbb{R}^3$

Panel 2

Length of C: $\int_C ds = \int_a^b \sqrt{(x')^2 + (y')^2}\, dt$

Work: $\int_C \vec{F} \cdot d\vec{r} = \int_C M\,dx + N\,dy + P\,dz$

Surface area $\iint_S dS = \iint_R \sqrt{1 + f_x^2 + f_y^2}\, dA$ if surface $S: f(x,y) = z$

$$\mathrm{curl}(\vec{F}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ M & N & P \end{vmatrix} = \begin{pmatrix} P_y - N_z \\ -(P_x - M_z) \\ N_x - M_y \end{pmatrix}$$

$\mathrm{div}(\vec{F}) = M_x + N_y + P_z$

$\mathrm{grad}(f) = \langle f_x, f_y, f_z \rangle$

Panel 3

$$\int_C ds = \int_a^b \sqrt{(x')^2+(y')^2}\,dt \qquad C: r(t) = \langle x(t), y(t)\rangle, \quad t\in[a,b]$$

$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{(x')^2+(y')^2}\,dt$$

$$\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))\,x'(t)\,dt$$

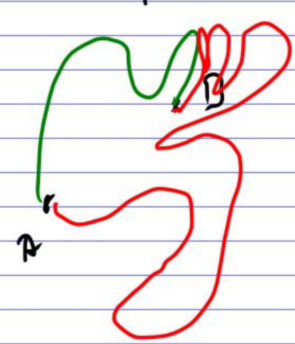
$$\iint_S g(x,y,z)\,dS = \iint_R g(x,y,f(x,y))\sqrt{1+f_x^2+f_y^2}\,dA$$

$z = f(x,y)$

Panel 4

$\int_C \vec{F}\,d\vec{r}$ independent of path if

$\int_{C_1} F\,dr = \int_{C_2} F\,dr$ for paths C_1, C_2 starting at A and ending at B



Fund. Thm: $\int_C \vec{F}\,d\vec{r} = f(B) - f(A)$

C from A to B

f is potential of F

Green's Thm: $\oint_C \vec{F}\,d\vec{r} = \iint_A \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dA$ if C closed

Panel 5

$$\iint_A f(x,y)\,dA = \lim_{n\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{n} f(x_i,y_j)\,\Delta x_i \,\Delta y_j$$

$$= \iint_{\text{bounds}} f(x,y)\,dx\,dy \quad \text{(Fubini)}$$

dy dx

$\text{curl}(\underbrace{\text{curl}(F)})$

$\text{curl}(F) = \langle xy,\ xy^2 - x,\ yx - z \rangle$

$\text{curl}(\quad)$

Panel 6

$F = \langle 6xy^2 - 3x^2,\ 6x^2y + 3y^2 - 7 \rangle$

$f_x = 6xy^2 - 3x^2 \rightsquigarrow f = 3x^2y^2 - x^3 + C(y)$

$f_y = \cancel{6x^2}y + C'(y) = \cancel{6x^2}y + 3y^2 - 7 \Rightarrow C = y^3 - 7y + c$

$\Rightarrow f = 3x^2y^2 - x^3 + y^3 - 7y + c$

Panel 7

Find the surface integral $\iint_S x-2y+z\,dS$, where S is the surface $z=10-2x+2y$ such that x is between 0 and 2 and y is between 0 and 4.

$$\iint_S x-2y+z\,dS = \iint_R \left(x-2y+\left(\underset{z}{f(x,y)}\right)\right)\sqrt{1+f_x^2+f_y^2}\,dA$$

$f(x,y)=z=10-2x+2y$

$f_x=-2,\ f_y=2$

$$= \iint_R x-2y+(\underset{z}{10-2x+2y})\sqrt{1+4+4}\,dA$$

$$= 3\int_0^2\int_0^4 x-\cancel{2y}+10-2x+\cancel{2y}\,dy\,dx$$

Panel 8

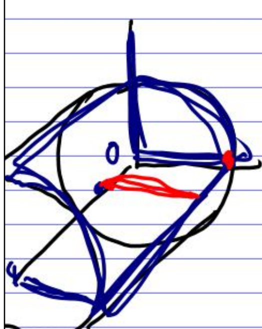
$\iint_S (x+z)dS$ where S is the first-octant portion of the cylinder $y^2+z^2=9$ between x = 0 and x = 4

$f(x,y)=z=\pm\sqrt{9-y^2}$

$$\iint_R \left(x+\sqrt{9-y^2}\right)\frac{3}{\sqrt{9-y^2}}\,dA$$

$f_x=0,\ f_y=-\dfrac{y}{\sqrt{9-y^2}}$

$\sqrt{1+f_x^2+f_y^2} = \sqrt{1+\dfrac{y^2}{9-y^2}} = \sqrt{\dfrac{9}{9-y^2}} = \dfrac{3}{\sqrt{9-y^2}}$



$$\int_0^4\int_0^3 \left(x+\sqrt{9-y^2}\right)\frac{3}{\sqrt{9-y^2}}\,dy\,dx$$