Panel 1

Integration Summary

Line integral $\int_C \_\_ \, d\_\_$

vector field

function:
- $\int f \, ds$
- $\int f \, dx$
- $\int f \, dy$

$\int_C \vec{F} \, d\vec{r}$:
- $\int_C M\,dx + N\,dy + P\,dz$
- conservative: $f(B) - f(A)$
- closed curve: $\iint_D \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \, dA$ Green's thm (2D)

Panel 2

② Evaluate the following integrals:

Green, closed curve: a) $\oint_\gamma (xy^2 + e^y)\,dx + (x^2 y + xe^y)\,dy$, γ = unit circle

Find f, (no path): b) $\int_{(-1,0)}^{(0,1)} (3x^2 + 2y)\,dx + (2x - 2y)\,dy = f(0,1) - f(-1,0)$

c) $\int_C 3x^2 - 7yx \, ds$, C line from $(0,1)$ to $(2,3)$ $\qquad \sqrt{(x')^2 + (y')^2}\,dt$

Manual, conservative? d) $\int_\gamma \langle y, z, x \rangle \cdot d\vec{r}$, γ line from $(1,1,1)$ to $(2,3,4)$

Manually, conservative? e) $\oint \langle y, z, x \rangle \, d\vec{r} = 0$, γ circle of radius 3

Panel 3

$$\oint_\gamma \underbrace{(xy^2+e^y)}_{M}\,dx + \underbrace{(x^2y+xe^y)}_{N}\,dy\ ,\ \gamma = \text{unit circle}$$

$$= \iint \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA = \iint (2xy+e^y)-(2xy+e^y)\,dA = 0$$

[F is conservative]

$$\int_{(-1,0)}^{(0,1)} (3x^2+2y)\,dx + (2x-2y)\,dy = f(0,1) - f(-1,0) = -1+1 = 0$$

$$f_x = 3x^2+2y \Rightarrow f = x^3 + 2xy + \cancel{C(y)}\ -y^2+c$$

$$f_y = 2x + C'(y) = 2x-2y,\quad \Rightarrow C' = -2y,\quad C = -y^2+c$$

Panel 4

$$\int_C 3x^2 - 7yx\, ds\ ,\ C \text{ line from } (0,1) \text{ to } (2,3)$$

$$\ell(t) = \langle 0,1\rangle + t\langle 2,2\rangle,\quad t\in[0,1]$$

$$= \langle \underset{x}{2t},\ \underset{y}{1+2t}\rangle$$

$$\int_0^1 3(2t)^2 - 7(1+2t)(2t)\sqrt{2^2+2^2}\,dt\ ,\ ds = \sqrt{(x')^2+(y')^2}\,dt$$

Panel 5

$\int_\gamma \langle y,z,x\rangle \cdot d\vec{r} = \int \langle y,z,x\rangle \cdot \langle dx,dy,dz\rangle$ line $(1,1,1)$ to $(2,3,4)$

$\ell(t) = \langle 1,1,1\rangle + t\langle 1,2,3\rangle =$

$\int y\,dx + z\,dy + x\,dz =$ $\quad = \langle \underset{x}{1+t}, \underset{y}{1+2t}, \underset{z}{1+3t}\rangle \quad , t\in[0,1]$

$$\int_0^1 (1+2t)\cdot dt + (1+3t)\,2dt + (1+t)\,3dt$$

$\oint \langle y,z,x\rangle\, d\vec{r}$, γ circle of radius 3 in xy plane

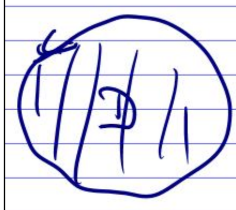
$$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ y & z & x \end{vmatrix} = \langle 1$$ not conservative!

$r(t) = \langle 3\cos(t), 3\sin(t), 0\rangle_{2\pi}$

$$\oint_C \langle y,z,x\rangle\, d\vec{r} = \int_0^{2\pi} y\,dx + z\,dy + x\,dz = \int_0^{2\pi} -9\sin^2(t) + 0 + 3\cos(t)\,0 = \int_0^{2\pi} -9\sin^2(t)\,dt$$

Panel 6

Prove $\oint_C x\,dy$ = area of interior of C = area(D)



$$\oint_C x\,dy = \oint_C \overset{M}{0}\,dx + \overset{N}{x}\,dy = \iint_D \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dA =$$

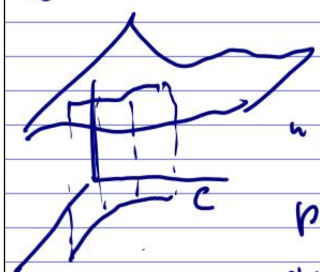
$$= \iint_D 1 - 0\,dA = \text{area}(D)$$

Panel 7

$y = f(x)$ a curve $r(t) = \langle t, f(t) \rangle$

$$\int_C ds = \int_a^b \sqrt{1+(f')^2}\, dt \quad \text{(length)}$$

$$\int_C g(x,y)\, ds = \int_a^b g(t, f(t)) \sqrt{1+(f')^2}\, dt$$



= int. of that portion of g over curve $y = f(x)$

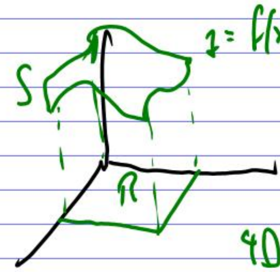
$z = f(x,y)$ is surface

surf. area

$$\iint_S dS = \iint_R \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

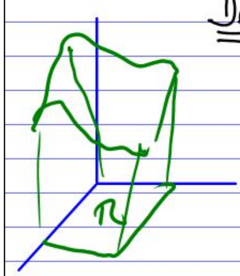
$$\iint_S g(x,y,z)\, dS =$$

$$= \iint_R g(x,y,f(x,y)) \sqrt{f_x^2 + f_y^2 + 1}\, dA$$



integral of that portion of 4D surface over S

Panel 8



Def: Suppose surface S is defined by $z = f(x,y)$, $(x,y) \in R$. Then

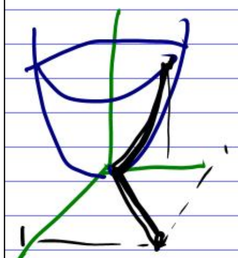
$$\iint_S g(x,y,z)\, dS = \iint_R g(x,y,f(x,y)) \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

$\iint_S g(x,y,z)\, dS$ is the integral of that portion of $w = g(x,y,z)$ over surface S

where S is a surface defined on R

Panel 9

Ex: $\int_\gamma x^2+y^2\,ds$, $y=x$, $x\in[0,1]$



$$\int_0^1 x^2+x^2\sqrt{1+1}\,dt = \int_0^1 \sqrt{2}\,2x^2\,dx = \frac{2}{3}\sqrt{2}$$

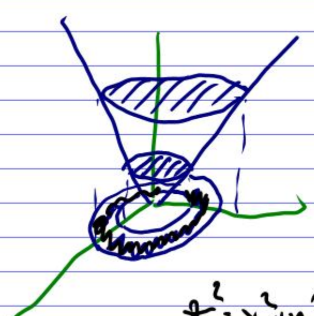
$$\iint x^2+y^2+z^2\,dS \quad z=x+y,\quad x,y\in[0,1]\times[0,1]$$

$$\iint_R x^2+y^2+(x+y)^2\sqrt{1+1+1}\,dA =$$

$$\int_0^1\int_0^1 (x^2+y^2+x^2+2xy+y^2)\sqrt{3}\,dx\,dy = \dots$$

Panel 10

Ex: Evaluate $\iint_S (x^2 z)\,dS$, S portion of $z^2=x^2+y^2$ between $z=1$ and $z=4$. surface integral



$$\iint_S g(x,y,z)\,dS =$$

$$\iint_R g(x,y,f(x,y))\sqrt{f_x^2+f_y^2+1}\,dA$$

$$z^2=x^2+y^2 \Rightarrow z=\sqrt{x^2+y^2}=f(x,y)$$

$$\iint_S x^2 z\,dS = \iint_R x^2\sqrt{x^2+y^2}\sqrt{f_x^2+f_y^2+1}\,dA = \text{HW}$$