

Panel 3

$f(x,y) = x^2 - xy + y^2$, $F(x,y) = \langle 2x - y, 2y - x \rangle$, $D = \{(x,y) : x^2 + y^2 \leq 1\}$,
 $C = \{(x,y) : x^2 + y^2 = 1, y \geq 0\}$, $\gamma_1(t) = \langle t, 0 \rangle$, $t \in [-1, 1]$, and $\gamma_2(t) = \langle t, \sin(\pi t) \rangle$, $t \in [-1, 1]$.

1. Sketch each object

① set in \mathbb{R}^2
 $F(1,2) = \langle 0, 3 \rangle$

Panel 4

Let $f(x,y) = x^2 - xy + y^2$, $F(x,y) = \langle 2x - y, 2y - x \rangle$, $D = \{(x,y) : x^2 + y^2 \leq 1\}$,
 $C = \{(x,y) : x^2 + y^2 = 1, y \geq 0\}$, $\gamma_1(t) = \langle t, 0 \rangle$, $t \in [-1, 1]$, and $\gamma_2(t) = \langle t, \sin(\pi t) \rangle$, $t \in [-1, 1]$.

- ~~b)~~ $\int_D f(x,y) dA$
- ~~c)~~ $\int_D f(x,y) ds$
- c) $\int_C f(x,y) ds$ ✓
- d) $\int_{\gamma_1} f(x,y) dx$ ✓
- e) $\int_{\gamma_2} f(x,y) dy$ ✓
- ~~f)~~ $\int_{\gamma_1} f(x,y) dr$
- ~~g)~~ $\int_{\gamma_1} F(x,y) dx$
- ~~h)~~ $\int_D F(x,y) dr$
- i) $\int_C F(x,y) dr$ ✓
- j) $\int_{\gamma_1} F(x,y) dr$ ✓
- k) $\int_{\gamma_2} F(x,y) dr$ ✓

Panel 5

Let $g(x) = x^2 + e^x$, $(f(x,y)) = xy \cdot e^{x^2}$, $(F(x,y)) = \langle xy, x^2 - y^2 \rangle$
 $C: r(t) = \langle 2t+1, t^3-3 \rangle, t \in [0,1], R = [0,1] \times [0,1]$. Setup:

① Area under g from 0 to 1
 $\int_0^1 x^2 + e^x dx$

② Length of curve C
 $\int_0^1 \sqrt{4 + 9t^4} dt$

③ Volume under f over R
 $\int_0^1 \int_0^1 xy e^{x^2} dx dy$

④ Surface area of f over R
 $\int_0^1 \int_0^1 \sqrt{(ye^{x^2} + x^2y)^2 + (xe^{x^2})^2 + 1} dx dy$

⑤ Work through \vec{F} along C
 $\int_C \vec{F} \cdot d\vec{r} = \int_C M dx + N dy = \int_0^1 xy dx + (x^2 - y^2) dy$
 $\int_0^1 (2t+1)(t^3-3) 2t dt + [(2t+1)^2 - (t^3-3)] t^3 dt$

⑥ Area of "curtain" over C under f
 $\int_C f ds$

Panel 6

$\vec{F} = \langle 3x^2 + 2y^2, 4xy + 3 \rangle$

$N_x = 4y \stackrel{?}{=} M_y = 4y$ ✓

$\nabla f = \langle P, Q \rangle = \vec{F} = \langle M, N \rangle$

$f_x = M: f_x = 3x^2 + 2y^2 \rightarrow f = \int (3x^2 + 2y^2) dx = x^3 + 2y^2x + C(y)$

$f_y = 4xy + C'(y) = 4xy + 3 \rightarrow C'(y) = 3y + 3$

$\Rightarrow f(x,y) = x^3 + 2y^2x + 3y + 3$ check: ∇f

Panel 7

$$\int_C x e^x dx,$$

C: $x = e^t$ from $(1,0)$ to $(e,1)$

$$r(t) = \begin{pmatrix} e^t \\ t \end{pmatrix}, t \in [0,1]$$

$x \quad y$

$$\int_0^1 e^+ \cdot e^+ e^+ dt = \int_0^1 e^{3t} dt = \frac{1}{3} e^{3t} \Big|_0^1 = \frac{1}{3} (e^3 - 1)$$

$$dx = x'(t) dt$$

Panel 8

Name: _____

Quiz 8

① Is $F(x,y) = \langle y \cos(x) + 2xy, \sin(x) + x^2 \rangle$ conservative?
If so, find its potential function.

b) How about $G(x,y) = \langle y^2 - x^2y, 2xy \rangle$?

Panel 9

② If C is given by $r(t) = \langle 2t, 3t-1 \rangle$, $t \in [0, 1]$, find:

a) $\int_C x^2 y \, ds$

b) $\int_C x^2 y \, dx$

c) $\int_C x^2 dx + y \, dy$