Panel 1

<u>Def:</u> Suppose $\vec{F}(x,y,z) = \langle M(x,y,z), N(x,y,z), P(x,y,z) \rangle$

Then $\quad \text{curl}(F) = \langle P_y - N_z, -(P_x - M_z), N_x - M_y \rangle$

$\text{div}(F) = M_x$

$$\vec{\nabla} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle = \langle \partial_x, \partial_y, \partial_z \rangle$$

$$\text{div}(F) = \vec{\nabla} \cdot \vec{F} = \langle \partial_x, \partial_y, \partial_z \rangle \cdot \langle M, N, P \rangle = \frac{\partial}{\partial x} M + \frac{\partial}{\partial y} N + \frac{\partial}{\partial z} P$$

$$\text{curl}(F) = \vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ M & N & P \end{vmatrix}$$

Panel 2

Ex $\vec{F} = \langle \underline{x^2yz}, \underline{xy^2z}, \underline{xyz^2} \rangle$

$$\text{div}(F) = 2xyz + 2xyz + 2xyz = \underline{\underline{6xyz}}$$

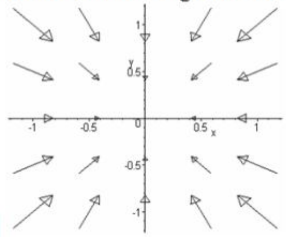
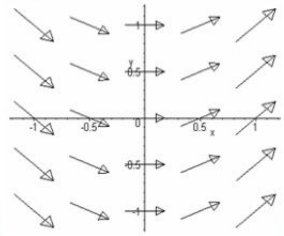
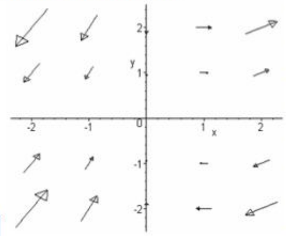
$$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ x^2yz & xy^2z & xyz^2 \end{vmatrix} =$$

$$= \underline{\underline{\langle xz^2 - xy^2, -(yz^2 - x^2y), y^2z - x^2z \rangle}}$$

Panel 3

Quiz 8 - Part 1

1. Below are three algebraic vector fields and three sketches of vector fields. Match them.

[A]  [B]  [C] 

(1) $F(x,y) = \langle xy, y(x-1) \rangle$

(2) $F(x,y) = \langle 1, x \rangle$

(3) $F(x,y) = \langle -x, -y \rangle$

Panel 4

2 Consider $\vec{F}(x,y,z) = \langle x^2+zy, y^2x^3-2xy, x-2zyx^3 \rangle$

a) Find $\text{div}(\vec{F})$

b) Find $\text{curl}(\vec{F})$

Panel 5

Def: A vector field $\vec{F}$ is conservative if there is a function $f(x,y,z)$ s.t. $\nabla f = \vec{F}$

f is called potential function of $\vec{F}$ "antiderivative"

Ex: Find vector field with potential
$$f(x,y,z) = x^2 - 3y^2 + 4z^2$$

$\vec{F} = \nabla f = \langle 2x, -6y, 8z \rangle$ is conservative with potential f.

Recall: a function f whose derivative is, say x^2, is $f(x) = \frac{1}{3}x^3$, i.e. f is antideriv. of x^2

Panel 6

Which of the following vector field(s) has as potential function $f(x,y,z) = x^2y^2z^2$

(a) $\vec{F} = \langle 2x, y, z \rangle$ $\quad f_x = 2xy^2z^2$

(b) $\vec{F} = \langle 2xy^2z^2 + x + y \rangle$

(c) $\vec{F} = \langle 2xy^2z^2, y, z \rangle$

(d) $\vec{F} = \langle 2xy^2z^2, x, y \rangle$

none of the above:

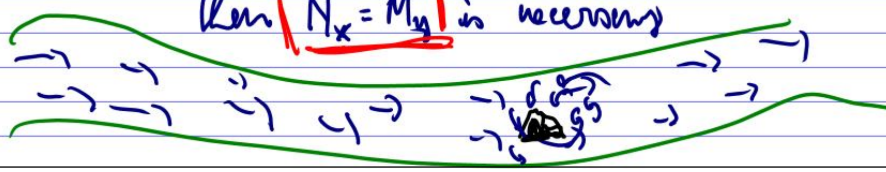
Panel 7

Suppose $\vec{F}$ is conservative, i.e. there is f with $\nabla f = (f_x, f_y, f_z) = \vec{F}$. Then find

$$\nabla \times (\nabla f) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ f_x & f_y & f_z \end{vmatrix} = \langle f_{zy} - f_{yz}, -(f_{zx} - f_{xz}), f_{yx} - f_{xy} \rangle$$

Theorem:

If a vector field F (all comps are 2x cont. diffble) wants to be conservative, then $\text{curl}(\vec{F}) = \nabla \times \vec{F} = \vec{0}$. If F is 2D-field then $\boxed{N_x = M_y}$ is necessary.



Panel 8

Which of the following vector fields is not conservative

(a) $F(x,y) = \langle \overset{M}{x}, \overset{N}{y} \rangle$ $\quad N_x - M_y = 0 - 0 = 0$ ✓

(b) $F(x,y) = \langle x^2 + y^2, 2xy \rangle$ ✓

(c) $F(x,y) = \langle e^x \cos(y), -e^x \sin(y) \rangle$ ✓

(d) $F(x,y) = \langle x^2 \cos(y), -y^2 \sin(x) \rangle$ ✗

$$N_x = -y^2 \cos(x) \neq M_y = -x^2 \sin(y)$$

Panel 9

$$F(x,y) = \langle M(x,y), N(x,y) \rangle$$
$$= \langle M(x,y,0), N(x,y,0), 0 \rangle$$

$$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ M & N & 0 \end{vmatrix} = (0-0, -(0-0), N_x - M_y)$$

Panel 10

Find potential function for $\vec{F} = \langle 3+2xy, x^2-3y^2 \rangle$ if exists

Is there one? $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$ $\quad N_x = 2x = 2x = M_y$ ✓

Know there is f with $\langle f_x, f_y \rangle = \langle 3+2xy, x^2-3y^2 \rangle$

$f_x = 3+2xy \Rightarrow f(x,y) = 3x + x^2y + C(y)$

$f_y = \cancel{x^2} + C'(y) = \cancel{x^2} - 3y^2 \Rightarrow C'(y) = -3y^2, \; C(y) = \underline{-y^3} + c$

$\Rightarrow \underline{f = 3x + x^2y - y^3 + c}$ $\quad$ Check $\nabla f = \langle \quad \rangle$ ✓

Panel 11

Find potential function for $\langle x^2 \overset{M}{\cos(y)}, -y^2 \overset{N}{\sin(x)} \rangle$

$N_x = -y^2\cos(x), \neq M_y = -x^2\sin(y)$

Panel 12

Find potential for $\vec{F} = \langle y^2, \underline{2xy + e^{3z}}, \underline{3y} e^{3z} \rangle$ if exists.

is curl$(F) = 0$? Too much trouble!

$f_x = y^2 \xrightarrow{\text{int}} f(x,y,z) = \underline{xy^2} + C(y,z)$

diff

$f_y = \cancel{2xy} + C_y = \cancel{2xy} + e^{3z} \xrightarrow{\text{int}} C_y = e^{3z} \Rightarrow C = ye^{3z} + D(z)$

diff $\quad f = xy^2 + ye^{3z} + D(z)$

Int.

$f_z = 3\cancel{y}e^{3z} + D'(z) = 3\cancel{y}e^{3z} \Rightarrow D'(z) = 0$

$\Rightarrow \underline{f(x,y,z) = xy^2 + ye^{3z} + c}$. Check $\nabla f =$