Panel 1

Last Time

Applications of Integration:

center of mass

surface area

Vector fields:

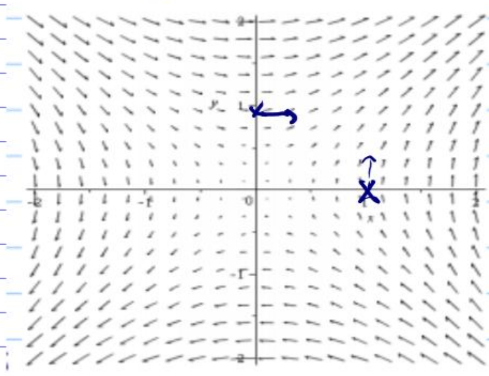
fieldplots

fieldplot3d

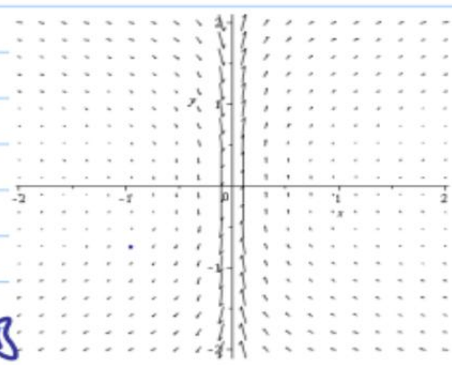
Maple

Panel 2

$\langle y, x \rangle$ $\langle y, \frac{1}{x} \rangle$



B



at $(1,0)$ → point up

$(0,1)$ → point right

Panel 3

$z = \sqrt{25 - x^2 - y^2} \iff x^2 + y^2 + z^2 = 25$

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

$S = \iint_R \sqrt{f_x^2 + f_y^2 + 1}\, dA$

$A = \pi r^2$

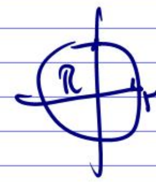
$S = 2\pi r$

$f_x = \frac{1}{2}(25 - x^2 - y^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{25 - x^2 - y^2}}$

$f_y = \frac{-y}{\sqrt{25 - x^2 - y^2}}$

$f_x^2 + f_y^2 + 1 = \frac{x^2 + y^2 + (25 - x^2 - y^2)}{25 - x^2 - y^2} = \frac{25}{25 - x^2 - y^2}$

$S = \iint_R \sqrt{\frac{25}{25 - x^2 - y^2}}\, dA = \int_0^{2\pi}\int_0^{5} \frac{5}{\sqrt{25 - r^2}}\, r\, dr\, d\theta$



Panel 4

$5 \int_0^{2\pi}\int_0^{5} \frac{r}{\sqrt{25 - r^2}}\, dr\, d\theta = 5 \int_0^{2\pi} -(25 - r^2)^{1/2}\Big|_0^{5}\, d\theta$

$= 5 \cdot 25^{1/2} \cdot 2\pi = 5^2 \cdot 2\pi$

Surface area of both halfs of sphere $= 4\pi(5)^2$ #

HW: Prove: $V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Fun "volume" of a ball in $\mathbb{R}^4, \mathbb{R}^5, \mathbb{R}^6, \ldots \mathbb{R}^n$

$x^2 + y^2 + z^2 + w^2 = R^2$

Panel 5

<u>Def:</u> Suppose $\vec{F}(x,y,z) = \langle M(x,y,z), N(x,y,z), P(x,y,z) \rangle$

Then $\underline{\text{curl}(\vec{F})} = \langle P_y - N_z, -(P_x - M_z), N_x - M_y \rangle$

$\underline{\text{div}(\vec{F})} = M_x + N_y + P_z$

Think of $\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$

$\text{grad}(f) = \underline{\nabla f} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$

$\underline{\text{div}(\vec{F}) = \nabla \cdot f} = \langle \partial_x, \partial_y, \partial_z \rangle \cdot \langle M, N, P \rangle = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

$\underline{\text{curl}(\vec{F}) = \nabla \times f} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ M & N & P \end{vmatrix} = \langle P_y - N_z, -(P_x - M_z), N_x - M_y$

Panel 6

2. Suppose that $F(x,y,z) = \langle x^3 z, x^2 z, xy \rangle$ is some vector field.

a) Find $div(F)$

$= 3x^2 z + 0 + 0 = \underline{3x^2 z}$

b) Find $curl(F)$

$\begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ x^3 z & x^2 z & xy \end{vmatrix} = \langle x - x^2, -(y - x^3), 2xz - 0 \rangle$

Panel 7

(7) If $f: \mathbb{R}^3 \to \mathbb{R}$ is a function and $\vec{F}: \mathbb{R}^3 \to \mathbb{R}^3$ is a vector field, which expression is meaningful:

~~curl (f)~~   curl (F) ✓ vector field

grad (f) ✓ gives vector field

div (F) ✓ gives function

curl (grad (f)) ✓

~~grad (F)~~

grad (div (F)) ✓

div (grad (f)) ✓

etc. HW

Panel 8

Show that $\nabla \cdot (f\vec{F}) = f(\nabla \cdot \vec{F}) + (\nabla f) \cdot \vec{F}$, $f: \mathbb{R}^3 \to \mathbb{R}$, $F = \langle M, N, P \rangle$

Proof:

Panel 9

Def: A vector field $\vec{F}$ is conservative if there is a function f s.t. $\nabla f = \vec{F}$

Ex: Find vector field with potential
$f(x,y,z) = x^2 - 3y^2 + 4z^2$

?

Well