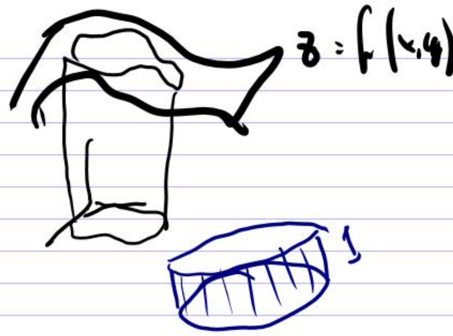
Panel 1

Applications of Integration



① $\iint_R f(x,y)\,dA$ volume

$\iint_R 1\,dA = \text{area}(R)$

② Mass of a Lamina

Suppose we have a lamina with density function $\rho(x,y)$

$$m = \iint_R \rho(x,y)\,dA$$

Panel 2

Lamina with Density function $\rho(x,y)$

Mass: $\iint_D \rho(x,y)\,dA = m$

Moments: $M_x = \iint_R y\, f(x,y)\,dA$

$$M_y = \iint_R x \cdot f(x,y)\,dA$$

Center of Gravity: $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{M_y}{m}, \quad \bar{y} = \frac{M_x}{m}$$

Panel 3

Find Center of Mass: first guess then confirm

uniform density

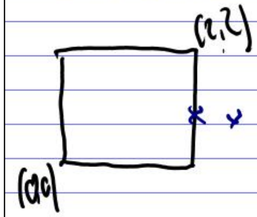
$$m = \int_0^2\int_0^2 k\,dA = k(2-0)(2-0) = 4k$$

$$M_x = k\int_0^2\int_0^2 y\,dA = 4k \qquad M_y = \int_0^2\int_0^2 x\,dA = 4k$$

$$\bar{x} = \frac{M_x}{m} = 1 \qquad \bar{y} = \frac{M_x}{m} = 1$$

Panel 4

Find Center of Mass: first guess then confirm

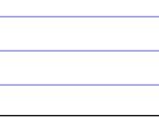


$$\rho(x,y) = x$$

$$m = \int_0^2\int_0^2 x\,dA = 4$$

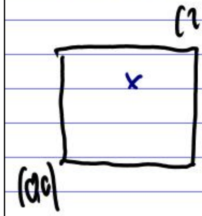
$$M_x = \iint_R xy\,dA = 4 \qquad M_y = \iint_R x^2\,dA = \frac{16}{3}$$

$$\bar{x} = 16/3 \cdot \frac{1}{4} = 4/3 \qquad \bar{y} = 1 = 4/4$$



Panel 5

Find Center of Mass: first guess then confirm



density y

HW

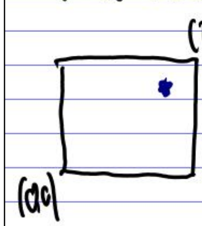
$m =$

$M_x =$ $\qquad$ $M_y =$

$\bar{x} = 1$ $\qquad$ $\bar{y} = \frac{4}{3}$

Panel 6

Find Center of Mass: first guess then confirm



$\rho(x,y) = x+y$ $\quad$ or xy

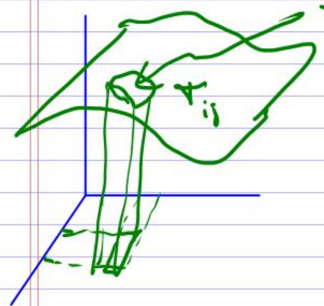
$m = \iint (x+y)\,dA = 8$

$M_x = \iint y(x+y)\,dA = \frac{28}{3}$ $\qquad$ $M_y = \iint x(x+y)\,dA = \frac{28}{3}$

$\bar{x} = \frac{28}{3} \cdot \frac{1}{8} = \frac{28}{24} = 1.$ $\qquad$ $\bar{y} = \frac{28}{24}$

Panel 7

Surface Area:



tangent plane $z = f_x \cdot (x - x_0) + f_y (y - y_0) + z_0$

$\text{area}(T_{ij}) = \sqrt{f_x^2 + f_y^2 + 1}\; dA$

surface area $\lim\limits_{n \to \infty} \sum\limits_{j}^{n} \sum\limits_{i}^{n} \text{area}(T_{ij})$

$$S = \iint_R \sqrt{f_x^2 + f_y^2 + 1}\; dA$$

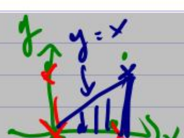
$dx\,dy$

$dy\,dx$

polar

Panel 8

Ex: Surface area of $z = x^2 + 2y$ above triangle $(0,1), (1,0)$, and $(1,1)$



$z = x^2 + 2y$

$f_x = 2x$

$f_y = 2$

$$S = \iint \sqrt{4x^2 + 4 + 1}\; dA$$

$$= \iint \sqrt{4x^2 + 5}\; dx\,dy \;?$$

$$= \int_0^1 \int_0^x \sqrt{4x^2+5}\; dy\,dx = \int_0^1 \sqrt{4x^2+5}\; x\,dx$$

$$= \int_0^1 x \sqrt{4x^2+5}\; dx \quad = \frac{2}{3} \cdot \frac{1}{8} (4x^2+5)^{3/2} \Big|_0^1$$

Panel 9

Ex: Surface area of $z = x^2 + 2y$ above square $[0,1] \times [0,1]$?

$$\int_0^1 \int_0^1 \sqrt{4x^2 + 5}\, dx\, dy$$

use $u = \tan(x)$ or Maple

$u^2 + 1 = \sec^2$

HW

Surface integrals are (almost) always difficult because of the $\sqrt{\quad}$! $\Rightarrow$ Maple to the Rescue!!!

Panel 10

Birds-Eye View so far

$f: \mathbb{R} \to \mathbb{R}$ calc I ✓

$f: \mathbb{R} \to \mathbb{R}^{2/3}$ space curves ✓

$f: \mathbb{R}^2 \to \mathbb{R}$ surfaces ✓

$f: \mathbb{R}^2 \to \mathbb{R}^2$

$\mathbb{R}^3 \to \mathbb{R}^3$

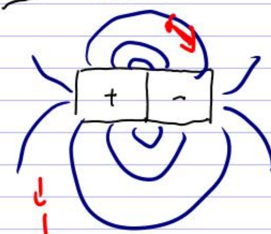
e.g. $f(x,y) = \langle xy, x+y \rangle$

Panel 11

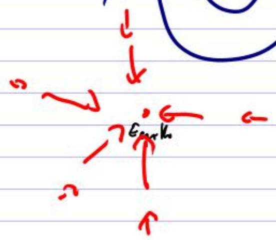
__Vector Fields:__ If for each point P in a region R there is a unique vector having initial point P, then the totality of such vectors is called a __vector field.__

__Ex:__ Flow of water

__Ex:__ Magnetic Field



__Ex:__ Gravity



Panel 12

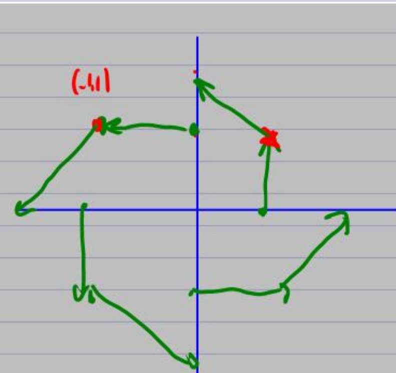
Mathematically, a vector field is given as:

$$F(x,y) = \langle M(x,y), N(x,y) \rangle$$

$$F(x,y,z) = \langle M(x,y,z), N(x,y,z), P(x,y,z) \rangle$$

__Ex:__ Describe $F(x,y) = \langle -y, x \rangle = -y\vec{i} + x\vec{j}$

(x,y)	$F(x,y)$
$(1,0)$	$\langle 0,1 \rangle$
$(1,1)$	$\langle -1,1 \rangle$
$(0,1)$	$\langle -1,0 \rangle$
$(-1,0)$	$\langle 0,-1 \rangle$
$(2,1)$	$\langle -1,2 \rangle$



Panel 13

Maple offers "fieldplot" and "fieldplot3d"

```
> with(plots);
> fieldplot([-y, x], x = -2..2, y = -2..2);
> fieldplot3d([-x/(x^2+y^2+z^2)^(3/2), -y/(x^2+y^2+z^2)^(3/2), -z/(x^2+y^2+z^2)^(3/2)], x = -2..2, y = -2..2, z = -2..2);
>
```

Panel 14

Def: If $r(x,y,z) = \langle x,y,z \rangle$ then $F(x,y,z) = \frac{c}{\|r\|^2}\vec{u}$ where $u = r/\|r\|$ is called inverse square field.

Ex: Describe inverse square field for $c = -1$.

$$F = \frac{-1}{\left(\sqrt{x^2+y^2+z^2}\right)^2} \cdot \frac{(x,y,z)}{\sqrt{x^2+y^2+z^2}} = \frac{-\langle x,y,z \rangle}{(x^2+y^2+z^2)^{3/2}}$$