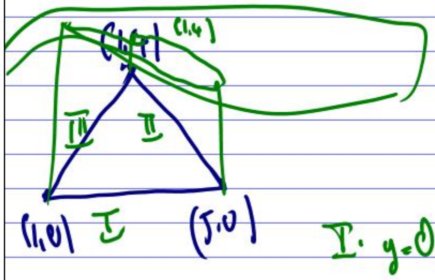
Panel 1

Last Time

Abs. extrema of: $f(x,y)=3+xy-x-2y$



$f_x: y-1=0$

$f_y: x-2=0$

$\underline{\underline{(2,1)}}$ is critical.

I: $y=0$, $x\in[1,5]$: $f(x,0)=3-x$

$f'(t)=-1\neq 0$ no criticals

II: $\ell(t)=(5,0)+t(-4,4)$, $t\in[0,1]$

$=(\underset{x}{5-4t},\underset{y}{4t})$

$\ell(\frac{1}{2})=\underline{\underline{(3,2)}}$ critical

$f(x(t),y(t))=3+(5-4t)(4t)-(5-4t)-2(4t)=$

$=3+20t-16t^2-5+4t-8t=$

$=-16t^2+16t-2$

$f'(t)=-32t+16=0 \Rightarrow t=\frac{1}{2}$

Panel 2

III: from $(1,0)$ to $(1,4)$: $\ell(t)=(1,0)+t(0,4)=$

$=(\underset{x}{1},\underset{y}{4t})$

$f(x,y)=\overset{3}{3}+\overset{4}{xy}-\overset{1}{x}-\overset{-8}{2y}=3+4t-1-8t=$

$f'(t)\neq 0$ no criticals

	$f(x,y)$	
$(2,1)$	1	
$(3,2)$	2	abs. max
$(1,0)$	2	
$(5,0)$	-2	abs. min
$(1,4)$	-2	

local extrema of $f(x,y)$

$f_x = y-1=0$

$f_y = x-2=0$

$(2,1)$ saddle point

$f_{xx}=0$,

$f_{yy}=0$

$f_{xy}=1$

$H=\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $D=-1$

Panel 3

$f(x,y) = (x+y) \cdot e^{xy^2}$. $D_{\vec{u}}(f)$ at $(0,2)$ in dir $(1,1)$

$f_x = 1 e^{xy^2} + \overset{2}{(x+y)} \overset{4}{y^2} e^{xy^2}$

$\Rightarrow \nabla f(0,2) = \langle 9, 1)$

$f_y = \underset{1}{1 e^{xy^2}} + \underset{2}{(x+y)} 2xy\, e^{xy^2}$

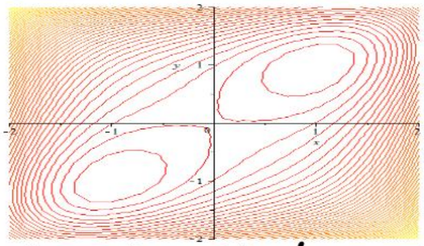
$\Rightarrow D_{\vec{u}}(f) = \nabla f(0,2) \cdot \frac{1}{\sqrt{2}}(1,1) = \frac{1}{\sqrt{2}}(9,1)\cdot(1,1) = \underline{\underline{\frac{10}{\sqrt{2}}}}$

Panel 4

Quiz

Name: ____________

①



Identify all critical points in this contour plot and classify them as max/min or saddle point(s).

② Consider $f(x,y) = x \sin(xy)$.

a) Find ∇f at $P(1,\pi)$

b) Find $D_{\vec{u}}(f)$ at $(1,\pi)$ in the direction $(4,5)$

Panel 5

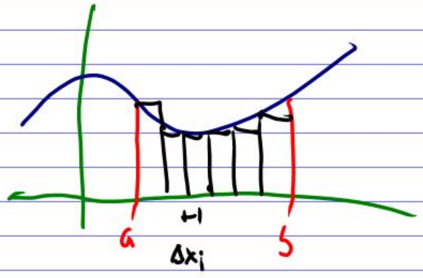
① Find all rel. extrema of $f(x,y) = x^2 - y^2 + 4xy$ and classify them

Panel 6

Skip. Method of Lagrange Multiplier

Panel 7

Integration: in $\mathbb{R}$.



antideriv.

$$\int_a^b f(x)\,dx \textcircled{=} F(b) - F(a)$$

(Fund. thm. of Calc)

= area under curve

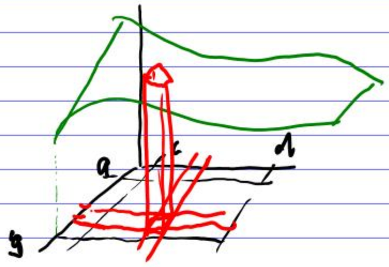
Not def.

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

limit of Riemann Sums

Panel 8

Integration in $\mathbb{R}^2$



Geometrically, volume under $f(x,y) = z$ (if $f \geq 0$)

$$\lim_{n,m\to\infty}\sum_{i=1}^{n}\sum_{j=1}^{m} f(x_i,y_j)\,\Delta x_i\,\Delta y_j \overset{\text{def}}{=}$$

$$\iint_{[a,b]\times[c,d]} f(x,y)\,dA$$

Panel 9

Fubini's Theorem (How to integrate in $\mathbb{R}^2$)

If $f(x,y)$ is continuous on $R=[a,b]\times[c,d]$

$$\iint_R f(x,y)\,dA = \underbrace{\int_a^b \underbrace{\int_c^d f(x,y)\,dy}_{}\,dx}_{} = \overbrace{\int_c^d \overbrace{\int_a^b f(x,y)\,dx}^{}\,dy}^{}$$

Ex: $\iint_R (x-3y^2)\,dA \quad , \quad R=[0,2]\times[1,2]$

$$=\int_1^2\int_0^2 (x-3y^2)\,dx\,dy = \int_1^2 \tfrac{1}{2}x^2-3y^2x\Big|_{x=0}^{x=2}\,dy = \int_1^2 2-6y^2\,dy =$$

$$=\int_0^2\int_1^2 (x-3y^2)\,dy\,dx = \int_0^2 xy-y^3\Big|_{y=1}^{2}\,dx = \underline{\underline{-11}}$$

HW

$$2y-2y^3\Big|_1^2 = (4-16)-(2-2) = \underline{\underline{-12}}$$