Panel 1

Last Time: How to find Relative Extremas

① ∇f gradient

② $\nabla f = 0$ solve system

③ $H = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$, $D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

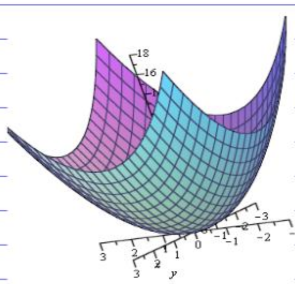
④ $D > 0$, $f_{xx} > 0$ min

$D > 0$, $f_{xx} < 0$ max

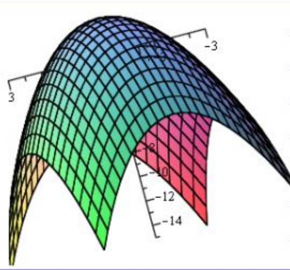
$D < 0$ saddle

$D = 0$ no info

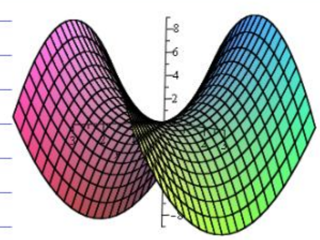
Panel 2



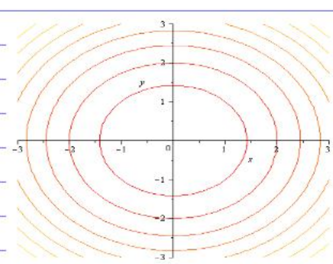
$f(x,y) = x^2 + y^2$

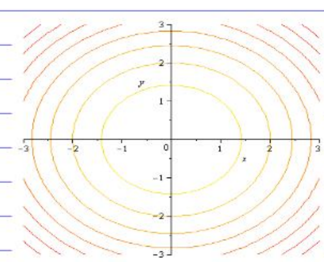


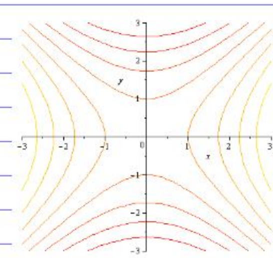
$f(x,y) = 1 - x^2 - y^2$



$f(x,y) = x^2 - y^2$







Panel 3

Gradient at $(2,-4)$: $(f_x, f_y) = \langle 24, 0\rangle$
(x y)

$f(x,y) := x^3 \cdot y + 12x^2 - 8y$

$$(x,y) \rightarrow x^3 y + 12x^2 - 8y$$

$f(1,1)$

$$5$$

$fx := diff(f(x,y), x)$

$$3x^2 y + 24x = f_x(2,-4) = 24$$

$fy := diff(f(x,y), y)$

$$x^3 - 8 = f_y(2,-4) = 0$$

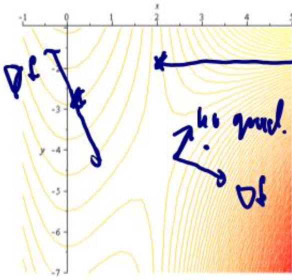
$solve(\{fx = 0, fy = 0\}, \{x, y\})$

$$\{x = 2, y = -4\}, \{x = 2\, RootOf(\_Z^2 + \_Z + 1), y = 4 + 4\, RootOf(\_Z^2 + \_Z + 1)\}$$

$diff(f(x,y), x\$2) \cdot diff(f(x,y), y, y) - diff(f(x,y), x, y)^2$

$$-9x^4$$

$contourplot(f(x,y), x = -1..5, y = -7..-1, contours = 100)$



Given: $D_{(1,1)} f(3,-4)$

$(\nabla f) \cdot \frac{1}{\sqrt{2}}(1,1)$

$\langle -9, 19\rangle \cdot \frac{1}{\sqrt{2}}(1,1) = \frac{1}{\sqrt{2}}(-9+19) = \frac{10}{\sqrt{2}} \approx$

max change

$\|\langle -9, 19\rangle\| = \ldots$

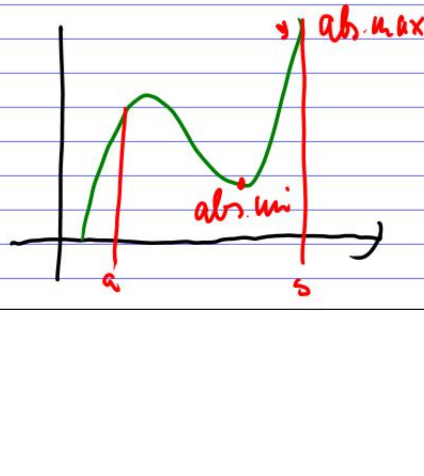
with(plots)

Panel 4

Absolute Max / Min:

Differences between absolute and relative extrema

Thm: If f is continuous on $[a,b]$, then f has max and min (abs). They can occur either at a critical pt. or at the endpoints a or b.



Panel 5

To find abs. max/min

If f is continuous on closed bounded set $D \subset \mathbb{R}^2$ (e.g. $[a,b]\times[c,d]$)

then f has abs. min + max

(1) Find $\nabla f = 0$

(2) ⇒ critical points

(3) critical points on boundary of D

(4) add endpoints if any

(5) table of values, pick largest, smallest

Panel 6

Ex: Find abs. extrema for $f(x,y) = x^2 - 2xy + 2y$ on $[0,3]\times[0,2]$, i.e. $0 \le x \le 3$ and $0 \le y \le 2$

(1) $f_x = 2x - 2y = 0$

$f_y: -2x + 2 = 0 \Rightarrow x = 1,\ y = 1$ is critical

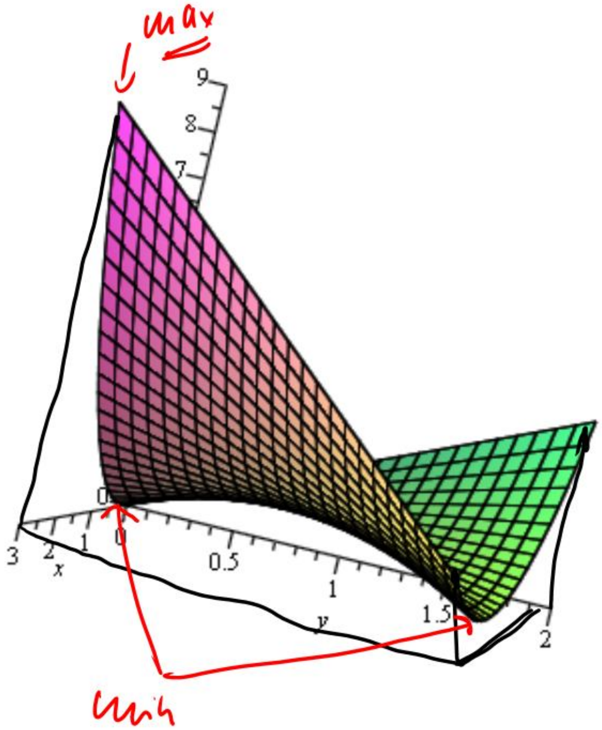
(3) I: $x=0$, $y \in [0,2]$, $f(0,y) = 2y$, $f'(y) = 2 \neq 0$ no critical

II: $x=3$, $y \in [0,2]$: $f(3,y) = 9 - 6y + 2y = 9 - 4y$ no

III: $y=0$, $x \in [0,3]$: $f(x,0) = x^2$, $f'(x) = 2x = 0$, $x = 0$ $(y=0)$

IV: $y=2$: $f(x,2) = x^2 - 4x + 4$, $f'(x) = 2x - 4 = 0$, $x = 2$ $(y=2)$

Panel 7



(x,y)	$z = f(x,y)$
$(1,1)$	1
$(2,2)$	0
$(0,0)$	0
$(0,2)$	4
$(3,0)$	9
$(3,2)$	2

abs. min

abs. max

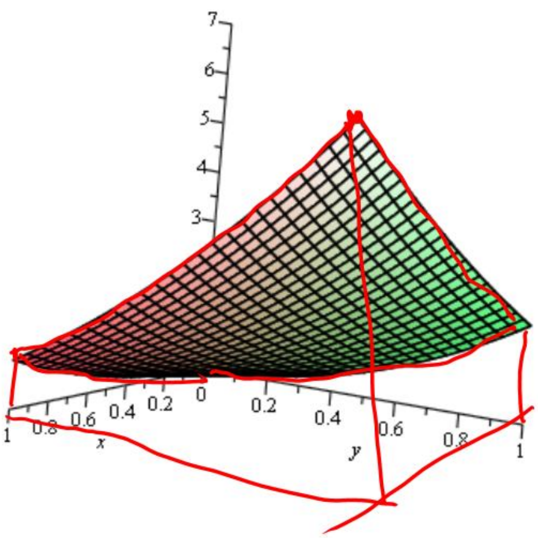
Panel 8

$f(x,y) = x^2 + 2y^2 + 4xy$ for $(x,y) \in [0,1] \times [0,1]$. $\to$ abs. extrema?

1 $\mathbb{R}^2$ problem $+ 4$ $\mathbb{R}$-problems!

Panel 9

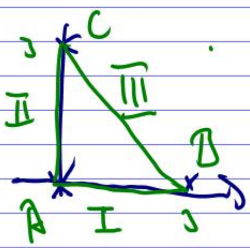
$f(x,y) = x^2 + 2y^2 + 4xy$



(x,y)	$f(x,y) = z$
$(0,0)$	0
$(0,1)$	2
$(1,0)$	1
$(1,1)$	7

Panel 10

Ex: Let $f(x,y) = 3xy - 6x - 3y + 7$. Find abs. extrema over triangle with corners $(0,0)$, $(3,0)$, and $(0,3)$



max/min inside $(\nabla f = 0)$

3 $\mathbb{R}$-problems.

II: $x = 0$: ?

I $y = 0$: ?

III $y = 3 - x$: $f(x,y) = C(x, 3-x) =$

$$= 3x(3-x) - 6x - 3(3-x) + 7$$

$C'(x) = 0 \Rightarrow x = ?$