Panel 1

Last time: Review of contour plots and surfaces.

limits: hope they are unequal along different paths

continuity: see limits: $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$

$f_x(x,y) = \frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$

$f_y(x,y) = \frac{\partial f}{\partial y} =$ –"–

$\frac{\partial^2 f}{\partial x \partial y} = \quad f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$

Panel 2

$f(x,y,z) = x y z^2 \tan(z)$

Panel 3

$f(x,y) = x^3y^2 + 5x^2 - 7y^2$ $f_x = 3x^2y^2 + 10x$, $f_y = 2x^3y - 14y$

$f(x,y) = x\sin(y) + e^{xy^2}$ $f_x: \sin(y) + x\cancel{\cos(y)} + y^2e^{xy^2}$

$f_y: x\cos(y) + 2xye^{xy^2}$

$f(x,y) = xe^{xy} + y\ln\frac{x}{y}$ $f_x: e^{xy} + yxe^{xy} + y/x - \cancel{y\ln(y)}$

$f(x,y,z) = xy\sin(yz)$

$\frac{\partial}{\partial y} y\ln\left(\frac{x}{y}\right) = \cancel{y}\cdot\frac{1}{\left(\frac{x}{y}\right)}\cdot\frac{1}{\cancel{y}}$

$f(x,y) = \frac{xy}{x^2+y^2}$

$f_y: x^2e^{xy} + \ln\left(\frac{x}{y}\right) - 1$

$f(x,y) = \frac{x\sin(x^2+y^2)}{1+y\cos(x)}$

$\frac{\partial}{\partial y} y\ln\frac{x}{y} = \ln\left(\frac{x}{y}\right) + \cancel{y}\cdot\frac{1}{\left(\frac{\cancel{x}}{\cancel{y}}\right)}\left(-\frac{x}{y^2}\right)$

$f(x,y) = \sqrt{1+(xy)^2}$

2nd order

Panel 4

$f(x,y) = \frac{xy}{x^2+y^2}$ $f_x = \frac{y(y^2-x^2)}{(x^2+y^2)^2}$ ✓

$f(x,y) = \frac{x\sin(x^2+y^2)}{1+y\cos(x)}$

$f(x,y) = \sqrt{1+(xy)^2} = (1+(xy)^2)^{1/2}$

2nd order

$f_y = \frac{x^2y}{\sqrt{1+(xy)^2}} = x^2y(1+(xy)^2)^{-1/2}$ $f_y = \frac{1}{2}(\quad)^{-1/2}\cdot 2(xy)\cdot x$

$f_y: \frac{x\sqrt{1+(xy)^2} - x^4y^2}{\sqrt{1+(xy)^2}\,(1+(xy)^2)}$ HW?

Panel 5

$xy\sin(yz)$ — f_x $y\sin(yz)$

f_y $x\sin(yz) + xyz\cos(yz)$

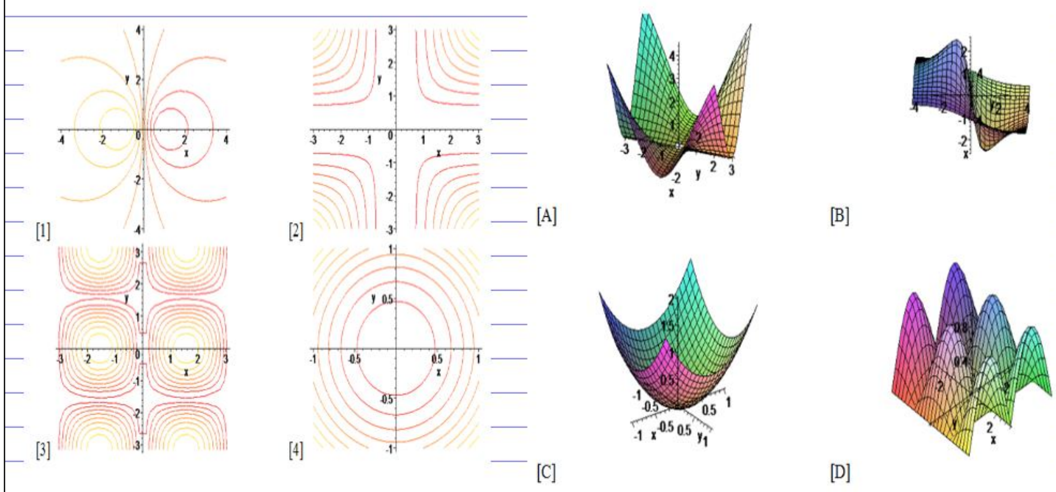
f_z $xy^2\cos(yz)$

Panel 6

Name: ____________

Quiz #5

① Match the surfaces on the right with the contour plots on the left.



Panel 7

#2) Find the limit if possible. Justify your argument.

$$\lim_{(x,y)\to(0,0)} \frac{3xy}{4x^2+5y^2}$$

#3) Find the indicated partial derivatives

a) $f(x,y) = xy + x^2y^3$

$f_x(x,y) =$

b) $f(x,y,z) = yz\sin(xz)$

$\frac{\partial f}{\partial z} =$

Panel 8

Partial derivatives frequently occur in Physics to describe laws of nature as PDEs (partial differential equations). For example: the Laplace PDE

$f(x) = f'(x)$ is ODE $\qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \iff u_{xx} + u_{yy} = 0$

is important in heat conduction and fluid flow.

Ex: Show that $f(x,y) = e^x \sin(y)$ satisfies the above PDE

$f_x = e^x \sin(y) \qquad f_y = e^x \cos(y)$

$f_{xx} = e^x \sin(y) \qquad f_{yy} = -e^x \sin(y)$

$f_{xx} + f_{yy} = 0$ ✓

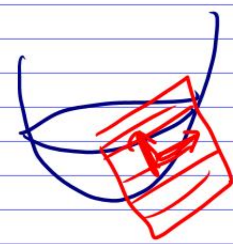
Panel 9

f_x = slope of f in x-dir
f_y = slope of f in y-dir
} f_x and f_y form a plane

Find the tangent plane to f at (x_0,y_0)

No cross products, because no vectors!

Want: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ or



$z = a(x-x_0)+b(y-y_0)+z_0 = \ell(x,y)$

$\ell(x,y)$ and $f(x,y)$ $\Rightarrow$ same tangents

$\Rightarrow \frac{\partial f}{\partial x} = a \qquad \frac{\partial f}{\partial y} = b$

$\boxed{z = f_x(x-x_0)+f_y(y-y_0)+z_0}$ is tangent plane

Panel 10

Equation of tangent plane to $f(x,y)$ at (x_0,y_0) is:

$$z = f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)+z_0$$

Ex: $f(x,y) = 2x^2+y^2$. Find tangent plane at $P(1,1,3)$

Panel 11

$f(x,y) = 2x^2 + y^2$ Tangent plane at $P(1,1,3)$ is:

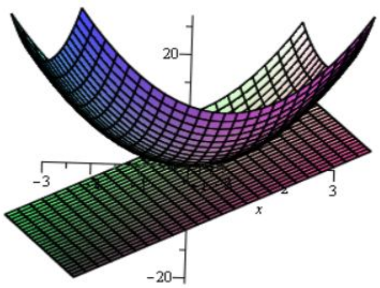
$$z = 4(x-1) + 2(y-1) + 3$$

Is P on the graph? $f(1,1)=3$ ✓

$f_x = 4x$ at $(1,1)$ ✓

$f_y = 2y$ at $(1,1)$ ✓

$plot3d\left(\{2\cdot x^2 + y^2, 4\cdot(x-1) + 2\cdot(y-1) + 3\}, x=-3..3, y=-3..3\right)$



Panel 12

The Chain Rule

$f: \mathbb{R} \to \mathbb{R},\ g: \mathbb{R} \to \mathbb{R} \Rightarrow \frac{d}{dx} f(g(x)) =$

next line

Chain Rule in $\mathbb{R}^2$:

$z = f(x,y),\ x = g(t),\ y = h(t)$

$$\Rightarrow \frac{\partial f}{\partial t} =$$

$z = f(x,y),\ x = g(s,t),\ y = h(s,t)$

$$\Rightarrow \frac{\partial f}{\partial s} =$$

$$\frac{\partial f}{\partial t} =$$