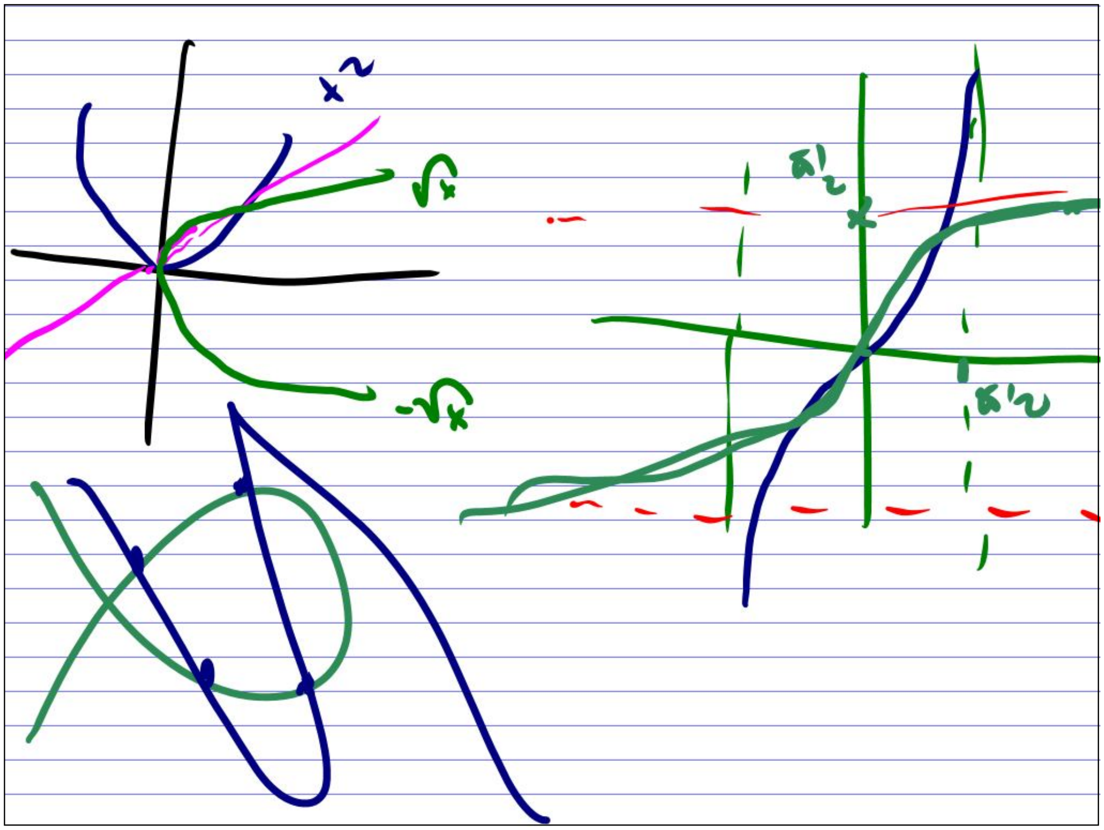
Panel 1



Panel 2

(t, t^2, t^3) and $(1+2t, 1+6t, 1+14t)$

$s \quad s \quad s$

$t = 1+2s$

$t^2 = 1+6s \qquad \Rightarrow (1+2s)^2 = 1+6s$

$t^3 = 1+14s$

$\cancel{1}+4s+4s^2 = \cancel{1}+6s$

$-2s+4s^2=0$

$2s(-1+2s)=0, \quad s = 0, \tfrac{1}{2}$

$\Rightarrow ((0),(1)), ((\tfrac{1}{2})(2))$

$s \quad t$

They intersect twice,
but at different times!
$\Rightarrow$ Don't crush!

Panel 3

Last Time:

Vector-valued functions:

Derivatives $\frac{d}{dt}\vec{r}(t) =$

Limits $\lim_{t \to t_0} \vec{r}(t)$

Integrals $\int_a^b \vec{r}(t)dt =$

Graphs of $\vec{r}(t)$

Length of curve:

Panel 4

Quiz #4

Name:

① Describe the curves given by

a) $\vec{r}(t) = \langle \cos(t), t, \sin(t) \rangle$

b) $\vec{r}(t) = \langle 5t, 1-t, 2t+1 \rangle$

② If $\vec{r}(t) = \langle t\sin(t), e^{t^2} \rangle$, find $\vec{r}'(t)$

Panel 5

③ If $\vec{r}(t) = \left\langle \frac{\sin(t)}{t}, \frac{\cos(t)-1}{t}, \frac{e^{t^2}-1}{t^2} \right\rangle$

find $\lim\limits_{t \to 0} \vec{r}(t)$

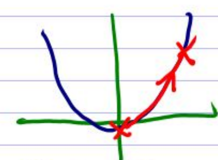
④ Find the length of $\langle 2t, t, 3t-1 \rangle$ for $0 \le t \le 1$

Panel 6

It is possible for one curve to have many different parametrizations:

Ex: $r_1(t) = \langle t, t^2 \rangle$, $t = 0, 1$

x $y = x^2$



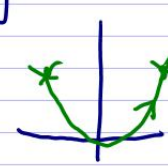
Same as: $\langle t^3, t^6 \rangle$

x $y = (t^3)^2 = x^2$

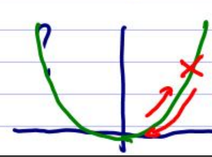
Looks the same, but timing is different!

Same as: $\langle \underbrace{\sin(t)}_{0..1}, (\sin(t))^2 \rangle$, $t \in [0, \pi/2]$

Different: $\langle t, t^2 \rangle$, $t \in -1$ to 1



$\langle t^2, t^4 \rangle$, $t = -1$ to 1



Panel 7

Find length of curve $r(t) = \langle 1+t, 2+2t, 3+3t \rangle$ for $t \in [0,1]$. Replace t by $2t$ and make sure to cover the same segment.

a) Interpret the effect of this

$\ell(t) = r(2t) = \langle 1+2t, 2+4t, 3+6t \rangle, \; t \in [0, \tfrac{1}{2}]$

~~slowed~~ speed the particle ~~down~~ up by 2

b) Find the length of this "new" line

length of r: $\int_0^1 \|r'\| \, dt = \int_0^1 \sqrt{1+4+9} \, dt = \sqrt{14}$

length of ℓ: $\int_0^{1/2} \|\ell'\| \, dt = \int_0^{1/2} \sqrt{4+16+36} \, dt = \cancel{2}\sqrt{14} \cdot \frac{1}{\cancel{2}} =$

Panel 8

Def: A curve $r(t)$ is called smooth if $\vec{r}'(t) \neq 0$, i.e. if the components of $\vec{r}'$ are not simultaneously zero.

Def. If $r(t)$ is a smooth curve then
$T(t) = r'(t)/\|r'(t)\|$ is called unit tangent

Ex: $r(t) = \langle t, t^2 \rangle$ find $T(t)$.

$\vec{r}'(t) = \langle 1, 2t \rangle$ is smooth (comp. are never zero simultaneously)

$T(t) = \langle 1, 2t \rangle \cdot \frac{1}{\sqrt{1+4t^2}}$

Panel 9

Theorem: $\vec{r}(t)$ space curve s.t. $\|\vec{r}(t)\| = 1$. Then
$\vec{r}(t) \cdot \vec{r}'(t) = 0$ i.e. $\vec{r}$ and $\vec{r}'$ are perp.

Thus: The vector $T'(t)$ is perp. to T
because $\|\vec{T}\| = 1$

The vector $\vec{N}(t) = T'(t) / \|T'(t)\|$ is called unit normal vector.
Summer!

Note: N is perp. to T

The vector $\vec{B} = \vec{N} \times \vec{T}$ is called binormal vector