Panel 1

Last Time

Intersections of (a) lines, (b) planes, (c) plane + line

Distances of

$P(x_0, y_0)$ and line $ax+by+c=0$ in $\mathbb{R}^2$

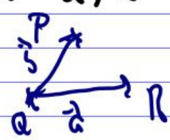
$$d = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$$

$P(x_0, y_0, z_0)$ and plane $ax+by+cz+d=0$ in $\mathbb{R}^3$

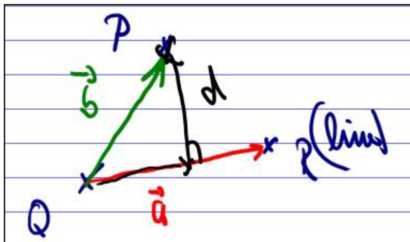
$$d = \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$$

$P(x_0, y_0, z_0)$ and line through Q, R in $\mathbb{R}^3$

$$d = \frac{\|a \times b\|}{\|a\|}$$



Panel 2



Hint: $a \times (b \times c) = (a \cdot c) b - (a \cdot b) c$

Problem: find normal vector.

$$d = \left\| \vec{b} - \text{proj}_{\vec{a}}(\vec{b}) \right\| = \left\| \vec{b} - \frac{\vec{a} \cdot \vec{b}}{\|a\|^2} \vec{a} \right\| =$$

$$\frac{1}{\|a\|^2} \left\| \|a\|^2 b - (a \cdot b)\vec{a} \right\| = \frac{1}{\|a\|^2} \left\| (a \cdot a) b - (a \cdot b)\vec{a} \right\| =$$

$$= \frac{1}{\|a\|^2} \left\| a \times (b \times a) \right\| = \frac{1}{\|a\|^{\cancel{2}}} \cancel{\|a\|} \, \|b \times a\| \cdot \sin(\theta) = \frac{\|b \times a\|}{\|a\|}$$

$\pi/2$

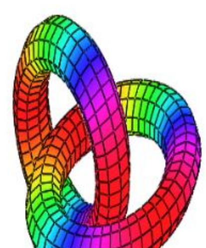
Panel 3

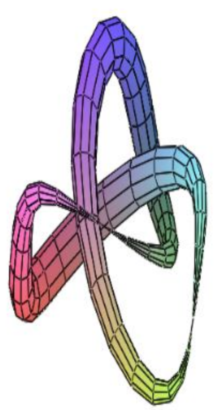
Chapter 12 Review

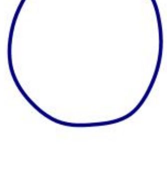
Started with $\mathbb{R}^3$, points, spheres, sheets
Vectors: add, subtr, length
Dot Product: projection
Cross Product
Lines
Planes
Mixed it up: distances, intersections!

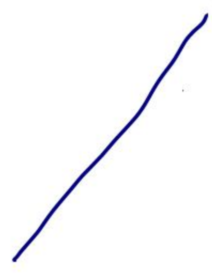
Panel 4

Space Curves









$\ell(t) = P + t\vec{v}$

(not planes)

Panel 5

Space Curves

Ex: $r(t) = \langle \sin(t), t^2+1, t-1 \rangle$
$= \langle 1+t, 2+3t, 3-t \rangle$

Def: $r(t) = \langle f(t), g(t), h(t) \rangle$ is a vector-valued function with component functions f, g, and h

Many concepts work as they should: If $\vec{r}(t)$ is vector-valued function then

Limit: $\lim_{t \to a} \vec{r}(t) = \langle \lim f(t), \lim g(t), \lim h(t) \rangle$

Derivative: $\vec{r}'(t) = \langle f', g', h' \rangle$

Integral: – ditto

Panel 6

Ex: $r(t) = \left\langle \frac{x^2 - 2x}{x}, \frac{\cos(x) - 1}{x}, \frac{\sin(x)}{x} \right\rangle \to 1$

Find $\lim_{t \to 0} r(t) = \langle -2, 0, 1 \rangle$

$\lim_{x \to 0} \frac{\cancel{x}(x-2)}{\cancel{x}}$, $\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$

$t^{-1/2}$

$r(t) = \left\langle t^3 + \frac{1}{\sqrt{t}} + \pi^2, t \cdot \ln(t), \frac{e^x \cos(x)}{\sin(x)} \right\rangle$

Find $r'(t) = \langle 3t^2 - \frac{1}{2} t^{-3/2}, 1 \cdot \ln(t) + t \cdot \frac{1}{t}, \frac{HW}{\sin^2(x)}$

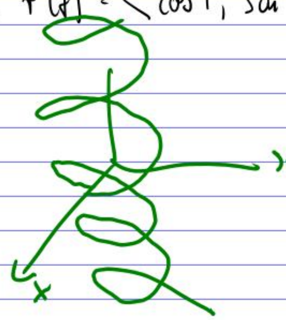
Panel 7

The problem: with vector-valued functions is to visualize them, and interprate the deriv. + integrals:

Ex: $\vec{r}(t) = \langle 1+t, 2+5t, -1+6t \rangle$ - describe graph

a line!

Ex: $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$ - describe graph



Panel 8

Sketch graph of

$r_1(t) = \langle (4+\sin(20t))\cos(t), (4+\sin(20t))\sin(t), \cos(20t) \rangle$

$r_2(t) = \langle (2+\cos(1.5t))\cos t, (2+\cos(1.5t))\sin(t) \quad \sin(1.5t) \rangle$

Maple: with(plots)

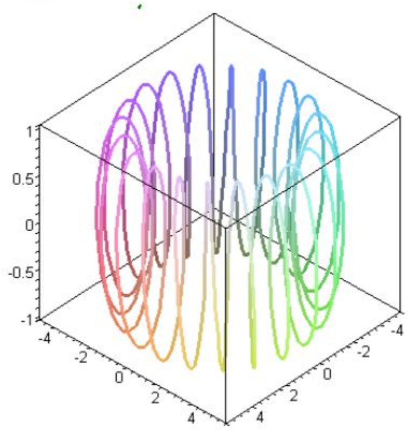
spacecurve ([x(t), y(t), z(t)], t = a..b)

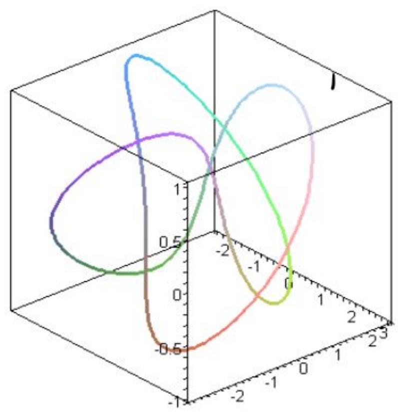
Panel 9

Sketch graph of

$r_1(t) = \langle (4+\sin(20t))\cos(t), (4+\sin(20t))\sin(t), \cos(20t) \rangle$

$r_2(t) = \langle (2+\cos(1.5t))\cos t, (2+\cos(1.5t))\sin(t) \quad \sin(1.5t) \rangle$

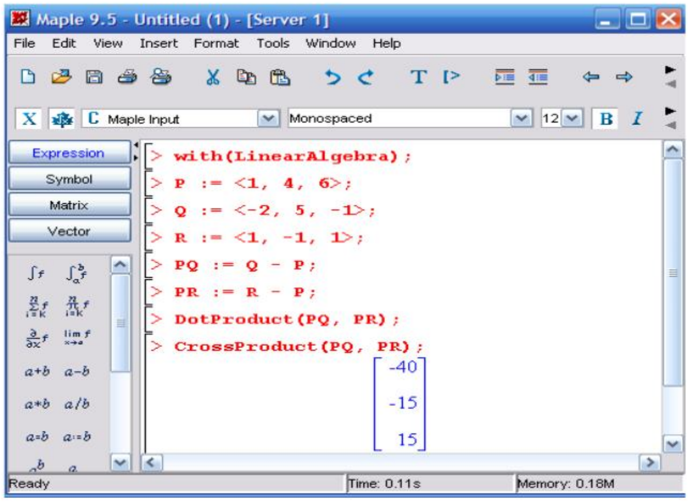


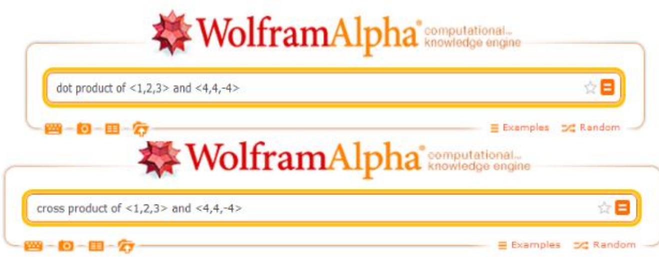


Panel 10

```
> with(plots);
> spacecurve( [(4+sin(20*t))*cos(t), (4+sin(20*t))*sin(t), cos(20*t)], t=0..2*Pi, numpoints=500);
> spacecurve( [(2+cos(1.5*t))*cos(t), (2+cos(1.5*t))*sin(t), sin(1.5*t)], t=0..4*Pi, numpoints=500);
> spacecurve([t, t^2, t^3], t=0..2);
```

Panel 11





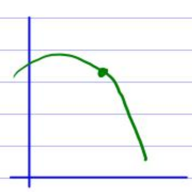
Panel 12

Derivatives of Space Curves aka Vector-valued functions

If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ and f, g, h are differentiable

then $\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$

0 0. 0



Ex: $\vec{r}(t) = \langle 1+t, t e^{-t}, \sin(2t) \rangle$

Find $\vec{r}(0)$ and $\vec{r}\,'(0)$

Compute $\vec{r}(0) \cdot \vec{r}\,'(0)$

$\vec{r}(0) = \langle 1, 0, 0 \rangle$

$\vec{r}\,'(t) = \langle 3t^2, e^{-t} - t e^{-t}, 2\cos(2t) \rangle$

$\vec{r}\,'(0) = \langle 0, 1, 2 \rangle$

$\Rightarrow \vec{r}(0) \cdot \vec{r}\,'(0) = 0$

Panel 13

Ex: Find equation of tangent line to $r(t) = \langle 2\cos t, \sin t, t \rangle$ at the point $P(0, 1, \pi/2)$

Is P on curve ??? $r(t) = \langle 0, 1, \pi/2 \rangle$! $\underline{t = \pi/2}$

$r'(t) = \langle -2\sin(t), \cos(t), 1 \rangle$

$r'(\frac{\pi}{2}) = \langle -2, 0, 1 \rangle$

$\ell(t) = \vec{P} + t\vec{v} = \langle 0, 1, \pi/2 \rangle + t\langle -2, 0, 1 \rangle$

Panel 14

Arc Length (Length of a curve)

If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ then $L = \int_a^b |r'(t)|\, dt$ is the length of the space curve.

Note: $\int_a^b \|r'(t)\|\, dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\, dt$

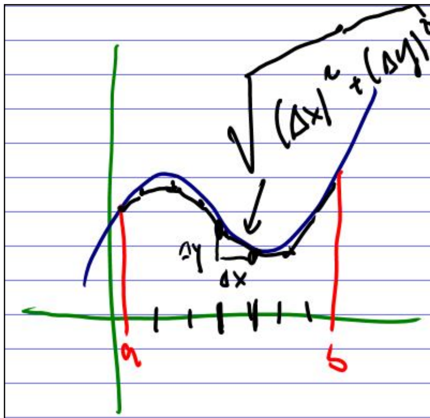
Ex: Find length of $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$, $t = 0$ to 2π

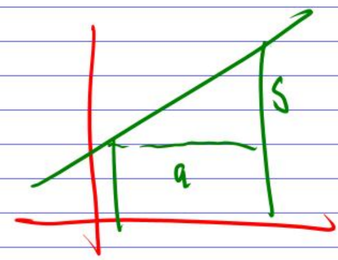
$$r'(t) = \langle -\sin(t), \cos(t) \rangle$$

$$\|r'\| = \sqrt{(-\sin(t))^2 + (\cos(t))^2} = 1$$

$$\Rightarrow L = \int_0^{2\pi} \|r'\|\, dt = \int_0^{2\pi} 1 = 2\pi$$

Panel 15





$$\sqrt{(\Delta x)^2+(\Delta y)^2} = \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2+\left(\frac{\Delta y}{\Delta t}\right)^2} \cdot \Delta t$$

$$L = \lim_{n\to\infty} \sum \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2+\left(\frac{\Delta y}{\Delta t}\right)^2}\, \Delta t = \int \sqrt{(x')^2+(y')^2}\, dt$$

$$\lim_{\Delta t\to 0} \frac{\Delta x}{\Delta t} = \lim_{t_2\to t_1} \frac{x(t_2)-x(t_1)}{t_2-t_1} = x' \qquad \lim_{t\to a} \frac{f(t)-f(a)}{t-a} = f'$$

Panel 16

Ex: Find length of $r(t) = \langle \cos(t), \sin(t), t \rangle$ from

$\underset{t=0}{(1,0,0)}$ to $(1,0,2\pi) \to t = 2\pi$

$$L = \int_0^{2\pi} \|r'\|^2\, dt = \sqrt{2}\, 2\pi$$

$$= \int_0^{2\pi} \| \langle -\sin(t), \cos(t), 1 \rangle \| \, dt =$$

$$= \int_0^{2\pi} \sqrt{(-\sin(t))^2 + (\cos(t))^2 + 1}\, dt =$$

$$= \int_0^{2\pi} \sqrt{2}\, dt = \sqrt{2} \int_0^{2\pi} 1\, dt = \sqrt{2} \cdot t \Big|_0^{2\pi} = \sqrt{2}(2\pi - 0) =$$

$$= \sqrt{2}\, 2\pi$$