Panel 1

Last Time

Line in $\mathbb{R}^{2 \text{ or } 3 \text{ or } n}$: $\ell(t) = P + t\vec{v}$ parametric eqn.

Plane in $\mathbb{R}^{3}$: $ax + by + cz + d = 0$ scalar eqn.

$\vec{n} = \langle a, b, c \rangle$ is normal vector to plane

$\mathbb{R}^2$: $ax + by + c = 0$

$\mathbb{R}^3$: $ax + by + cz + d = 0$

$\mathbb{R}^4$: $ax + by + Ez + dw + e = 0$ (hyperplanes)

Panel 2

Intersections:

$= \langle 1+1t, 2+t, 3+t \rangle$ P

Find intersection of $\ell(t) = \langle 1,2,3 \rangle + t\langle 1,1,1 \rangle$ and

$2x - y + z = 0$

$2(1+t) - (2+t) + (3+t) = 0 \Rightarrow t = \#$

Find intersection of $\ell(t) = \langle 0,1,0 \rangle + t\langle 1,0,1 \rangle$ and

$\ell(s) = \langle -1,-2,1 \rangle + s\langle 2,3,0 \rangle$ YES

$\ell(t) = \langle 0+t, 1+0, t \rangle$

Find intersection of $-x + y + z = 0$ and

$2x - y + z = 1$

$t = -1 + 2s$

$1 = -2 + 3s$

$t = 1 \Rightarrow s = 1$

They intersect in a line. That line is on plane, hence $\ell(t) = P + \vec{v}t$ with $\vec{v}$ perp to n.

Panel 3

Find intersection of (1) $-x+y+z=0$ and

(2) $2x-y+z=1$

They intersect in a line. That line is on plane, hence $l(t)=P+\vec{v}t$ with $\vec{v}$ perp to n_1

Also, $\vec{v}$ perp. to n_2 because l is part of plane 2

$\Rightarrow \vec{v}$ is parallel to $n_1 \times n_2$: $\begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = \langle 2,3,-1 \rangle$

$\Rightarrow l(t)=\underbrace{\langle 1,1,0 \rangle} + t\langle 2,3,-1 \rangle$. Most likely, line goes through xy-plane, i.e. $z=0$: $-x+y=0$

$2x-y=1$

$x=1, y=1$

Another point on l is $l(1)=\langle 3,4,-1 \rangle$

Panel 4

<u>Intersection of 2 Planes</u> : $l(t)=P+t\vec{v}$

line perp. to both planes

$\Rightarrow \vec{v}=\vec{n}_1 \times \vec{n}_2$

Find any point common to both planes:

Set $z=0$, solve for x,y

Panel 5

$x+y+z=0$ and $2x-y+z=1$

Panel 6

Name ____________

Quiz #3

(1) Find equations for:

a) a line through $P(3,-1,2)$ and $Q(0,1,1)$

b) a plane through $P(3,-1,2)$, $Q(0,1,1)$, $R(-1,1,2)$

Panel 7

(2) Is $P(0,1,2)$ on line $\ell(t) = \langle 2,3,0\rangle + t\langle -1,-1,1\rangle$?

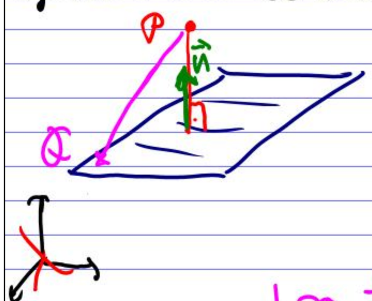
(4) Are $2x - y + 3z = 5$ and $3x + 6y - z = 0$ perpend.?

(5) Find intersection of $\ell(t) = \langle 1,1,2\rangle + t\langle 2,-1,3\rangle$ and
$2x - y + 3z = 21$

Panel 8

Distances

a) Distance between point P and plane $ax+by+cz+d=0$



Pick any point Q on plane

$\Rightarrow d = \|\mathrm{proj}_{\vec{n}} \vec{PQ}\|$

$$d = \frac{|\vec{PQ}\cdot\vec{n}|}{\|\vec{n}\|}$$

Panel 9

Ex: Find distance of $10x + 2y - 2z = 5$ to origin $P(0,0,0)$

1. Find any Q in the plane $Q(\frac{1}{2}, 0, 0)$
2. Find $\vec{PQ}$
3. $d = \|\text{proj}_{\vec{n}} \vec{PQ}\|$ or $Q(0, 0, -\frac{5}{2})$

$\vec{PQ} = \langle 1/2, 0, 0 \rangle$ $PQ = (0, 0, -5/2)$

$$d = \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|} = \frac{|\langle \frac{1}{2}, 0, 0 \rangle \langle 10, 2, -2 \rangle|}{\sqrt{108}} = \frac{5}{\sqrt{108}}$$

$$d = \frac{\langle 0, 0, -5/2 \rangle (10, 2, -2)}{\sqrt{108}} = 5/\sqrt{108}$$

Panel 10

Find distance between

a) $10x + 2y - 2z = 5$ and $P(1,1,1)$

b) $10x + 2y - 2z = 5$ and $x + y + z = 1$

c) $10x + 2y - 2z = 5$ and $5x + y - z = 1$

a) $5/\sqrt{108}$

$$d = \frac{|PQ \cdot n|}{\|n\|}$$

b) planes are not parallel $\Rightarrow$ $d = 0$

$\langle 10, 2, -2 \rangle$ and $\langle 1, 1, 1 \rangle$

pick any P on plane 1

c) $\langle 10, 2, -2 \rangle \parallel \langle 5, 1, -1 \rangle$ ✓

any Q on plane 2

Panel 11

We know the planes $10x + 2y - 2z = 6$ and $x + y + z = 1$ are not parallel. Thus, they intersect! Find intersection!

HW

Panel 12

Formula: The distance between $P(x_0, y_0, z_0)$ and plane $ax + by + cz + d = 0$ is $d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$

HW

Ex dist. between $P(1,1,1)$ and $\underbrace{10x + 2y - 2z - 5 = 0}$

before

$$d = \frac{|10 \cdot 1 + 2 \cdot 1 - 2 \cdot 1 - 5|}{\sqrt{108}} = \underline{\frac{5}{\sqrt{108}}}$$

Panel 13

Find a formula for the distance between $P(x_0, y_0)$ and a line $ax + by + c = 0$ in $\mathbb{R}^2$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

(very similar to $\mathbb{R}^3$ situation)

Panel 14

Distance Formulas

$P(x_0, y_0)$ and line $ax + by + c = 0$ in $\mathbb{R}^2$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$P(x_0, y_0, z_0)$ and plane $ax + by + cz + d$ in $\mathbb{R}^3$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$P(x_0, y_0, z_0)$ and line through Q, R in $\mathbb{R}^3$

$$d = \frac{\|a \times b\|}{\|a\|}, \quad \vec{a} = \vec{QR}, \quad \vec{b} = \vec{QP}$$

Panel 15

Still missing: dist. between 2 lines !!!

next time, Wed!