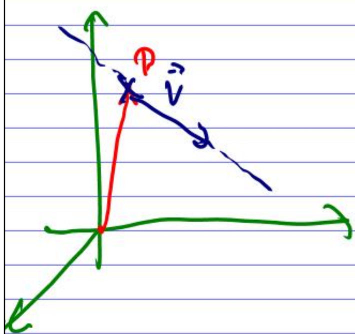
Panel 1

Last time

Dot Product: $\vec{a} \cdot \vec{b} = \|a\| \|b\| \cos\theta$

Cross Product: $\|\vec{a} \times \vec{b}\| = \|a\| \|b\| \, |\sin\theta|$

Parametric Equation of line:



$$\ell(t) = P + vt = \langle \underbrace{p_1 + tv_1}_{= x}, \underbrace{p_2 + tv_2}_{= y}, \underbrace{p_3 + tv_3}_{= z} \rangle$$

Panel 2

HW 2d) $i + j = \langle 1, 1, 0 \rangle$, $j + k = \langle 0, 1, 1 \rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \langle 1, -1, 1 \rangle$$

$i = \langle 1, 0, 0 \rangle$ $j = \langle 0, 1, 0 \rangle$, $k = \langle 0, 0, 1 \rangle$

$$\ell(t) = \langle 2, 1, 0 \rangle + t \langle 1, -1, 1 \rangle$$

Panel 3

$\ell(t) = \langle 2,3\rangle + t\langle 2,6\rangle$

① $P(2,3)$, $Q(4,9)$ (or $Q(6,15)$)

$m = \frac{9-3}{4-2} = \frac{6}{2} = \underline{\underline{3}}$

$\ell(t) = P + t\vec{v} = \vec{P} + t\langle v_1, v_2\rangle$ $\quad y = mx + b$, $m = v_2/v_1$

$= \langle p_1 + tv_1, p_2 + tv_2\rangle$

$x \quad y$

$P(p_1, p_2)$, $Q(p_1+v_1, p_2+v_2)$ $\boxed{m = \frac{v_2}{v_1}}$

Panel 4

$\ell(t) = \langle -1-2t, 3t, 1+4t\rangle$ $P\langle -3, 3, 5\rangle$ is on the line?

$\ell(t) = P$: $-1-2t = -3$ ✓

$3t = 3 \to t = 1$ $\quad$ YES!

$1+4t = 5$ ✓

Panel 5

Suppose 2 lines are $\ell_1(t) = \langle 1+t, -2+3t, 4-t \rangle$

$\ell_2(t) = \langle 2t, 3+t, -3+4t \rangle$

a) Are they parallel? check if $\vec{v}_1 \parallel \vec{v}_2$

$\langle 1, 3, -1 \rangle \parallel \langle 2, 1, 4 \rangle$ No!

b) Do they intersect? If they did, they'd have a common pt.

$\ell_1(t) = \ell_2(s) \Leftrightarrow 1+t = 2s \Rightarrow t = 2s-1$

$-2+3t = 3+s \quad -2+6s-3 = 3+s$

$4-t = -3+4s \quad 5s = 8, \; s = \frac{8}{5}$

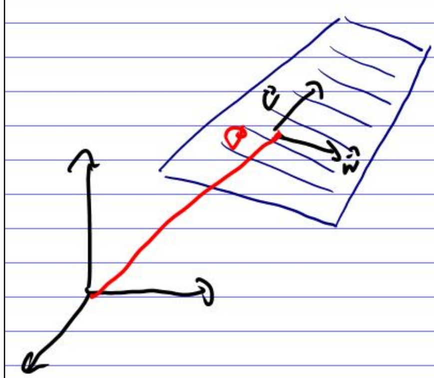
$\Rightarrow t = \frac{11}{5}$

No

Not good!

Panel 6

Planes in $\mathbb{R}^3$



A plane in $\mathbb{R}^3$ is uniquely determined by

- 3 points
- 2 vectors + 1 point

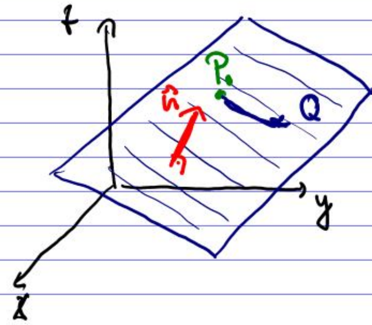
$p(t) = P + t\vec{v} + s\vec{w}$ (parametric equation)

- one vector perp. to plane, 1 point!

Panel 7

Suppose a plane goes through $P_0(x_0, y_0, z_0)$ and has normal vector $\vec{n} = \langle a, b, c \rangle$

⊥ perp. to plane



If $Q(x, y, z)$ is on the plane

$\Rightarrow \vec{PQ} \cdot \vec{n} = 0$

$\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

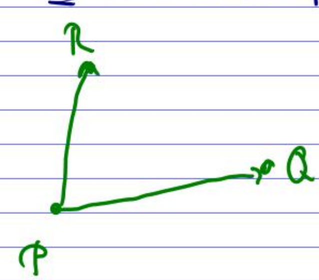
or $\quad ax + by + cz + d = 0$

$\uparrow (ax_0 + by_0 + cz_0)$

Panel 8

Scalar equation of Plane through $P(x_0, y_0, z_0)$ with normal vector $\vec{n} = \langle a, b, c \rangle$ is $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Ex: Plane through $P(1, 3, 2)$, $Q(3, -1, 6)$ and $R(5, 2, 0)$



$\vec{PQ} \times \vec{PR} \overset{\boxed{\text{say}}}{=} \langle 3, -2, 1 \rangle$

$3(x - 5) - 2(y - 2) + (z - 0) = 0$

$3(x - 1) - 2(y - 3) + (z - 2) = 0$

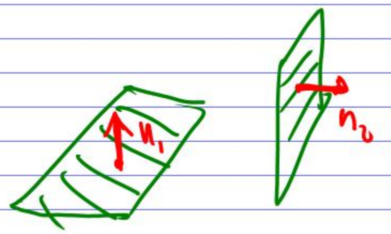
or: $3x - 2y + z + d = 0$

To find d, subst. P (or Q, R) and solve for d.

Panel 9

Scalar equation of plane through $P(x_0, y_0, z_0)$ with normal vector $\vec{n} = \langle a, b, c \rangle$ is $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

Ex: Angle between planes $x+y+z=1$ and $x-2y+3z=1$



is same as angle between the 2 normal vectors, i.e

$n_1 = \langle 1,1,1 \rangle$

$n_2 = \langle 1,-2,3 \rangle$

$\cos(\theta) = \dfrac{n_1 \cdot n_2}{\|n_1\| \|n_2\|} = \ldots$ ✓

Panel 10

Consider the plane $\underline{2x} + \underline{3y} + 4z = 12$

a) Find any point on plane $(0,0,3)$ or $(6,0,0)$ or

b) Find point with $x = 42$, $y = 7$ on the plane

$2 \cdot 42 + 3 \cdot 7 + 4z = 12 \Rightarrow z = \_\_$

c) Is $P(\overset{x}{0}, \overset{y}{2}, \overset{z}{-1})$ on the plane?

$2 \cdot 0 + 3 \cdot 2 + 4(-1) \neq 12$ Nope

d) Does $\ell_1(t) = \langle 1,2,0 \rangle + t\langle 3,-1,1 \rangle$ intersect the plane? How about $\ell_2(t) = \langle 0,4,0 \rangle + t\langle 2,-4,2 \rangle$?

HW

Panel 11

Does $\ell_1(t) = \langle 1,2,0\rangle + t\langle 3,-1,1\rangle$ intersect the plane? How about $\ell_2(t) = \langle 0,4,0\rangle + t\langle 2,-4,2\rangle$?

(*) $2x+3y+4z=12$ parallel to ℓ_1?

$$\langle 2,3,4\rangle \cdot \langle 3,-1,1\rangle \neq 0$$

Line is parallel to plane if $\vec{n}\cdot\vec{v}=0$

$$\ell(t) = \langle \underset{x}{1+3t}, \underset{y}{2-t}, \underset{z}{t}\rangle$$

$$2(1+3t)+3(2-t)+4(t)=12$$
$$2+6t+6-3t+4t=12$$
$$7t=4 \qquad t=4/7$$

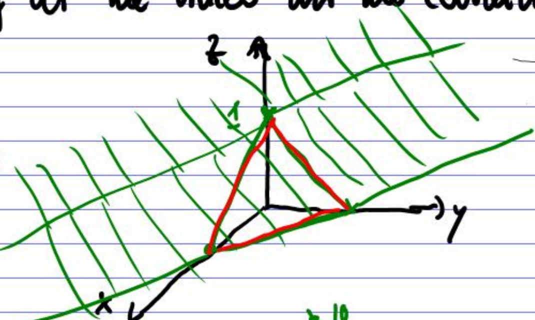
$\Rightarrow \ell(4/7)=$ point of intersection!

Panel 12

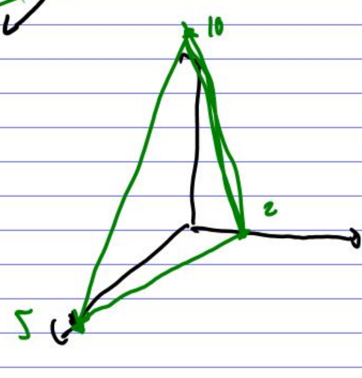
<u>Graphing Planes</u>: Planes can be visualized by looking at the traces in the coordinate planes.

$x+y+z=1$

$x+y=1, \ y=1-x$



$2x+5y+z=10$



$3x+y+2z=6$

Panel 13

Intersections:

Find intersection of $\ell(t) = \langle 1,2,3\rangle + t\langle 1,1,1\rangle$ and
$2x - y + z = 0$ ✓

Find intersection of $\ell(t) = \langle 0,1,0\rangle + t\langle 1,0,1\rangle$ and
$\ell(s) = \langle -1,-2,1\rangle + t\langle 2,3,0\rangle$ ✓

Find intersection of $-x + y + z = 0$ and
$2x - y + z = 1$?