Panel 1

Last Time

Dot Product. $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$= \|\vec{a}\| \|\vec{b}\| \cos(\theta)$

Projection of $\vec{b}$ onto $\vec{a}$: $proj_{\vec{a}}(\vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\|a\|^2} \vec{a}$

Length of $proj_{\vec{a}}(\vec{b})$: $comp_{\vec{a}}(\vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\|a\|}$

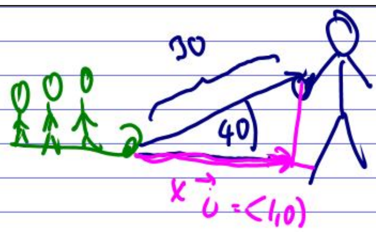
Cross Product. $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$:

perp. to both $\vec{a}, \vec{b}$

$= \langle a_2 b_3 - a_3 b_2, -(a_1 b_3 - a_3 b_1), a_1 b_2 - a_2 b_1 \rangle$

$= \|\vec{a}\| \|\vec{b}\| \sin(\theta) \vec{u}$

Panel 2



pull with 30 lbs for 80 feet.

$x = 30 \cos(40)$

$W = xd = (30 \cos(40)) \cdot 80$

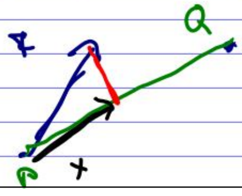
$\vec{F} = 30 \langle \cos(40), \sin(40) \rangle$ polar coords.

$proj_{\vec{i}}(F) = \frac{F \cdot i}{\|i\|^2} i = 30 \cos(40) \langle 1, 0 \rangle$

$= \frac{F \cdot PQ}{\|PQ\|} \cdot PQ$

$= \frac{F \cdot PQ}{\|PQ\|^2} \cdot PQ \cdot \|PQ\|$

$8i - 6j + 9k = F$ from $P\ (0,10,8)$ to $Q\ (6,12,20)$



$F = \langle 8, -6, 9 \rangle$

$PQ = \langle 6, 2, 12 \rangle$

$W = x \|PQ\| = proj_{PQ}(F) \cdot \|PQ\|$

Panel 3

Quiz #2

Name: ______________

(1) Find the dot product $\langle 3,-2,1\rangle \cdot \langle 1,2,2\rangle$

(2) Which vector is perpendicular to $\langle 3,-2,1\rangle$:

a) $\langle 1,1,1\rangle$

b) $\langle 2,4,2\rangle$

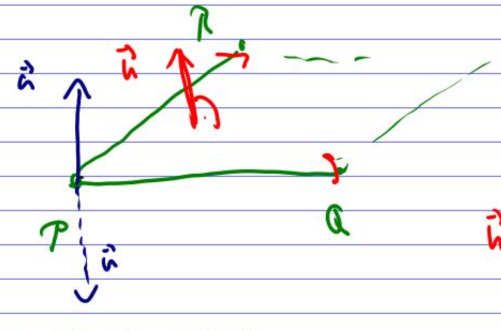
Panel 4

(3) Find the projection of $\langle 3,-1,-2\rangle$ onto $\langle 3,3,1\rangle$

(4) Find the cross product $\langle 3,-2,1\rangle \times \langle 1,2,2\rangle$

Panel 5

Ex: Find vector perpendicular to the plane through $P(1,4,6)$, $Q(-2,5,-1)$, and $R(1,-1,1)$



$PQ = \langle -3, 1, -7 \rangle$

$PR = \langle 0, -5, -5 \rangle$

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 1 & -7 \\ 0 & -5 & -5 \end{vmatrix} = \langle -5-35, -15, 15 \rangle$$

Right-Hand Rule: sweep fingers from a to b, thumb points in dir. of $\vec{a} \times \vec{b}$

Panel 6

Q: Find area of triangle $P(1,4,6)$, $Q(-2,5,-1)$ and $R(1,-1,1)$

$= \frac{1}{2} \| PQ \times PR \|$

Fact: $\|\vec{a} \times \vec{b}\|$ is the area of parallelogram a, b

Q: Is the crossproduct commutative? No!

Fact: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

Q: Is crossproduct associative? No!

Fact: $(a \times b) \times c \neq a \times (b \times c)$

Panel 7

Cross out the expressions that do not make sense. For the rest, is the answer a vector or a scalar?

$a \cdot (b \times c)$	$(a \cdot b) \times c$
$(a \cdot b) c$	$(a \times b) + c$
$a \times (b \cdot c)$	$(a \cdot b) + c$
$\|a\| (b \cdot c)$	$(a \cdot b) + c$
$a \times (b \times c)$	$(a \times b) \cdot (c \times d)$
$a \cdot (b + c)$	$\|a\| (b \times c)$
$(a \times b) + c$	$\|a \times b\|$

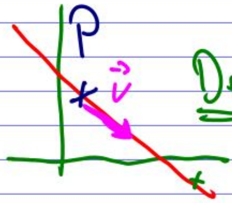
Panel 8

Of Lines and Planes

Line: $y = mx + b$ No good! $\mathbb{R}^2$ only, but

$z = mx + ny + b$ is NOT a line! in $\mathbb{R}^3$

Excludes vertical lines $(x = 5)$



Def. $\ell(t) = P + t\vec{v}, \; t \in \mathbb{R}$ is called parametric equation of line through P in direction of $\vec{v}$

Panel 9

Def: If $P(x_0, y_0, z_0)$ is a point on a line, and $\vec{v} = \langle a, b, c \rangle$ is the direction of the line, then the (parametric) equation of the line is:

$$\ell(t) = P + t\vec{v}$$

Panel 10

Ex: Find equation of a line

a) through $(5,1,3)$ and parallel to $\vec{v} = \langle 1,4,-2 \rangle$

$$\ell(t) = (5,1,3) + t\langle 1,4,-2 \rangle$$

b) through $\overset{P}{(1,2,3)}$ and $\overset{Q}{(4,1,1)}$

$$\ell(t) = (1,2,3) + t\langle 4-1, 1-2, 1-3 \rangle =$$

$$= \langle 1,2,3 \rangle + t\langle 3,-1,-2 \rangle$$

$$= \langle 1+3t, 2-t, 3-2t \rangle$$

Panel 11

Line through $(1,2)$ and $(4,7)$

$$\ell(t) = (1,2) + t\langle 3,5\rangle$$

vert. line through $(5,7)$

$$\ell(t) = (5,7) + t\langle 0,1\rangle$$

Panel 12

Ex: At what point does $\langle 2,4,-3\rangle + t\langle 1,-5,4\rangle$ intersect the xy-plane

$$\langle 2+t,\ 4-5t,\ -3+4t\rangle$$

$$-3+4t = 0 \Rightarrow t = 3/4$$

Ex: Is $P(1,2,3)$ on line $\langle 2,4,-3\rangle + t\langle -1,-2,3\rangle$

Find a) any point on $\langle 1,2,3\rangle + t\langle 2,-1,0\rangle$

b) with x-comp. 42: