Panel 1

MOOCs e.g. "Mooculus" - Google

https://mooculus.osu.edu/

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Panel 2

Last Time: Dot Product

$\vec{V} \cdot \vec{W} = \langle V_1, V_2, V_3 \rangle \cdot \langle W_1, W_2, W_3 \rangle$

$= V_1W_1 + V_2W_2 + V_3W_3$

Properties

(1) $\vec{V} \cdot \vec{W}$ is a scalar (#) not a vector

(2) $\vec{V} \cdot \vec{V} = \|V\|^2$

(3) $\cos(\theta) = \dfrac{V \cdot W}{\|V\| \cdot \|W\|}$

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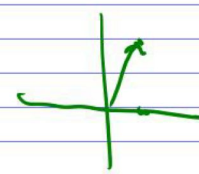
Panel 3

We also talked about "Directional angles" of $\vec{v}$

a) with x-axis $(\vec{i} = \langle 1,0,1 \rangle)$: $v_1/\|v\|$

b) with y-axis $(\vec{j} = \langle 0,1,0 \rangle)$: $v_2/\|v\|$

c) with z-axis $(\vec{k} = \langle 0,0,1 \rangle)$ $v_3/\|v\|$



$f(x) = x^2$. Tangent line at $(2,4)$ has slope: $f'(x) = 2x$,

$f'(2) = \underline{4}$

Two vectors are parallel if they have the same direction.

From $(2,4)$ to $(3,8)$, i.e. $\vec{w} = (3-2, 8-4) = \underline{(1,4)}$

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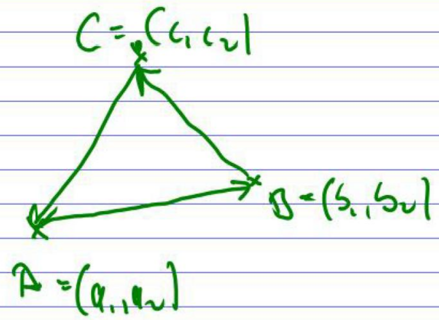
Panel 4



$\vec{AB} = \langle \cancel{b_1} - \cancel{a_1}, \cancel{b_2} - \cancel{a_2} \rangle$

$\vec{BC} = \langle \cancel{c_1} - \cancel{b_1}, \cancel{c_2} - \cancel{b_2} \rangle$

$\vec{CA} = \langle \cancel{a_1} - \cancel{c_1}, \cancel{a_2} - \cancel{c_2} \rangle$

$+ \quad \langle 0, 0 \rangle$

2 vectors are perp: if $v \cdot w = 0$ (or orthogonal)

2 vectors are parallel if $\vec{v} = c \cdot \vec{w}$

$(2,3)$ is parallel to a) $(8,12)$

~~b)~~ $(6,10)$

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Panel 5

$\langle -3, 9, 6\rangle \parallel \langle 4, -12, -8\rangle$

$\vec{v}$ $\vec{w}$

$4\cdot\vec{v} = \langle -12, 36, 24\rangle$, $-3\vec{w} = \langle -12, 36, 24\rangle$

$\Rightarrow 4\vec{v} = -3\vec{w}$

$\vec{v} = -\frac{3}{4}\vec{w}$

$-3\cdot c = 4 \Rightarrow c = -\frac{4}{3}$

$-\frac{4}{3}\cdot 9 = -12$

$-\frac{4}{3}\cdot 6 = -8$

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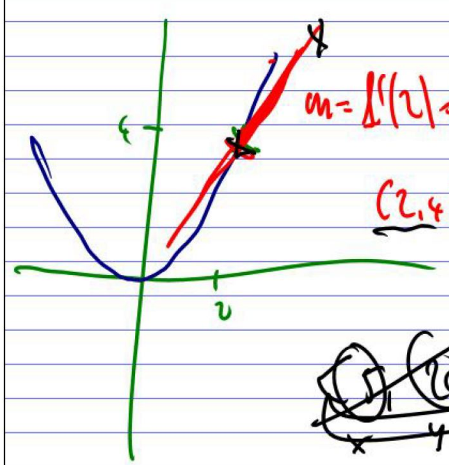
Panel 6



$m = f'(2) = 4$

$(2,4)$ to $(3,8)$: $\langle 3-2, 8-4\rangle = \langle 1,4\rangle$

~~$(5, 20)$~~ ~~$\frac{y}{x}$ = slope.~~

$\langle 3,7\rangle$ to $\langle 4,11\rangle \Rightarrow \langle 1,4\rangle$

or $\langle 2,8\rangle$, $\langle 3,12\rangle$, $\langle 4,16\rangle$

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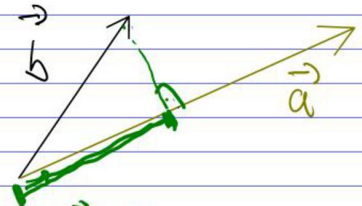
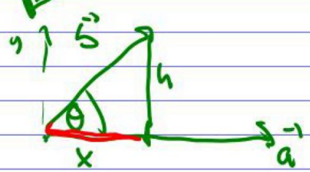
Panel 7

General Question: take two vectors $\vec{a}$ and $\vec{b}$. How much of $\vec{b}$ goes in the direction of $\vec{a}$?





$x =$

$\frac{x}{\|b\|} = \cos(\theta) \Rightarrow x = \|b\| \cos(\theta) =$

$= \cancel{\|b\|} \frac{\vec{a}\cdot\vec{b}}{\|a\| \cancel{\|b\|}}$.

Length: $\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}$

Vector: $\frac{\vec{a}\cdot\vec{b}}{\|a\|} \cdot \frac{a}{\|a\|} = \frac{\vec{a}\cdot\vec{b}}{\|a\|^2}\vec{a}$ $\left(\left\| \frac{a\cdot b}{\|a\|^2} \cdot a \right\| = \frac{|a\cdot b|}{\|a\|^2} \|a\| \right)$

Panel 8

Projection Formula: $\text{proj}_{\vec{a}}(\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\vec{a}$

Ex: Find length and direction of projection of $\vec{b} = \langle 1,1,2 \rangle$ onto $\vec{a} = \langle -2,3,1 \rangle$

$\text{proj}_{\vec{a}}(b) = \frac{a\cdot b}{\|a\|^2} a$

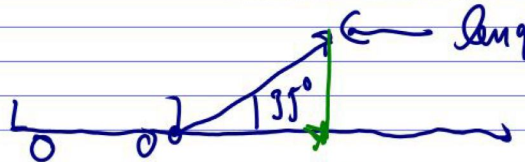
$a\cdot b = \langle 1,1,2 \rangle \cdot \langle -2,3,1 \rangle = -2+3+2 = 3$

$\|a\| = \sqrt{14}$

$= \frac{3}{14} \cdot \langle -2,3,1 \rangle$

Panel 9

Application: A wagon is pulled a distance of 100 m by a constant force of 70 N, applied to a handle held at 35°. Find work done by $\vec{F}$.



← length is 70

1st comp. of $\vec{F}$

$$\text{proj}_{i}(\vec{F}) = \frac{\vec{F}\cdot i}{\|i\|^2}\cdot i = (\vec{F}\cdot i)\, i = F_1 \cdot i$$

$$= \|\vec{F}\|\cos(35^\circ)$$

$\vec{F} = \langle F_1, F_2 \rangle$, $\quad \vec{F}\cdot i = \langle F_1, F_2\rangle \cdot \langle 1, 0 \rangle = F_1$

$\|i\| = 1$

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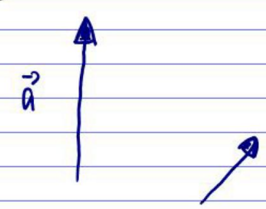
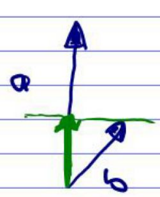
Panel 10

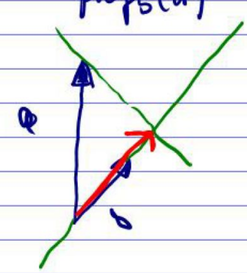
Picture Problems



find $\text{proj}_{\vec{a}}(\vec{b})$



$\text{proj}_{\vec{b}}(\vec{a})$



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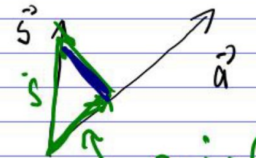
Panel 11

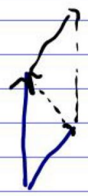
It looks like green vector is perp. to $\vec{a}$.



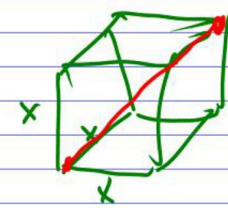
$\text{proj}_{\vec{a}}(b)$

Prove it: green/blue vector is

$$\left(\text{proj}_{\vec{a}}(b) - \vec{b}\right)\cdot \vec{a}$$



$$\frac{a\cdot b}{\cancel{\|a\|^2}}\cancel{a\cdot a} - \vec{a}\cdot\vec{b} = 0$$



Panel 12

So: Add / Subtract vector → vectors ✓

Dot product of vectors → scalar ✗

Cross Product: $\vec{a} = \langle a_1, a_2, a_3\rangle$, $\vec{b} = \langle b_1, b_2, b_3\rangle$

$$\vec{a}\times\vec{b} = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1\rangle$$

? ? ?

Panel 13

How to memorize the cross product:

$$\vec{a} \times \vec{b} = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \langle \overset{i}{a_2b_3 - a_3b_2},\ \overset{j}{-(a_1b_3 - a_3b_1)},\ a_1b_2 - a_2b_1 \rangle$$

Ex: $\langle 1,3,4 \rangle \times \langle 2,7,-5 \rangle =$

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Panel 14

$\langle 1,0,-2 \rangle \times \langle 0,2,-3 \rangle$

$$\begin{matrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 2 & -3 \end{matrix} \quad \langle \overset{i}{0(-3) - 2(-2)},\ \overset{j}{1(-3) - 0(-2)},\ \overset{k}{1(2) - 0(0)} \rangle$$

$$\langle 4, -3, 2 \rangle$$

$\langle 1,0,0 \rangle \times \langle 0,1,0 \rangle$

$$\begin{vmatrix} i & j & k \\ 1, & 0, & 0 \\ 0, & 1, & 0 \end{vmatrix} \quad \langle \overset{i}{0\cdot0 - 0\cdot1},\ \overset{j}{-1\cdot0 - 0\cdot0},\ \overset{k}{1\cdot1 - 0\cdot0} \rangle$$

$$\langle 0, 0, 1 \rangle$$

$\langle 1,0,-2 \rangle \cdot \langle 0,2,-3 \rangle$ 6

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Panel 15

Properties: (1) $\vec{a} \times \vec{a} = 0$

(2) $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

(3) $\|\vec{a} \times \vec{b}\|$ is area of parallelogram $\vec{a}, \vec{b}$

(4) $\|\vec{a} \times \vec{b}\| = \|a\| \|b\| |\sin(\theta)|$

Recall: $a \cdot b = \|a\| \|b\| \cdot \cos(\theta)$

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