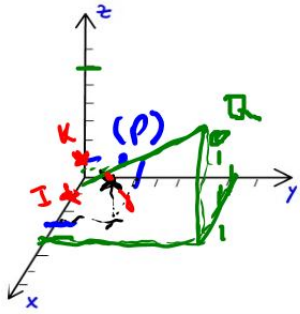
Panel 1

Last time

Review of Calc 1 + 2 ✓

Panel 2

Plot the following Points.



P(3,2,1) - use blue

Q(4,5,6) - use green

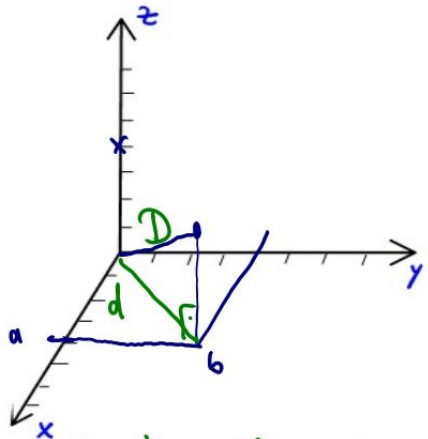
I = (1,0,0)

j = (0,1,0)

K = (0,0,1)

Panel 3

Distance in $\mathbb{R}^3$



$P(a, b, c)$

Distance between origin and P

$d^2 = a^2 + b^2$

$D^2 = d^2 + c^2 = a^2 + b^2 + c^2$

Distance to origin is $D = \sqrt{a^2 + b^2 + c^2}$

Dist. between $P(x_1, y_1, z_1)$ and $P(x_2, y_2, z_2)$ is

$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Panel 4

$P(3,2,1)$ and $Q(4,5,6)$

Find distance to origin and distance P to Q

$D(O,P) = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

$D(O,Q) = \sqrt{16 + 25 + 36}$

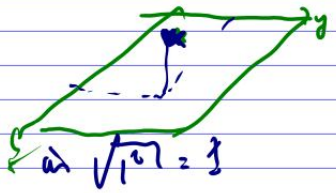
$D(P,Q) = \sqrt{(4-3)^2 + (5-2)^2 + (6-1)^2}$

Dist. of P to xy-plane? To x-axis?

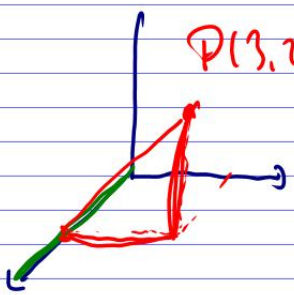
Ex: $P(3,2,1)$ dist. to xy-plane

$Q(x, y, 0)$

Minimize $D(P,Q) = \sqrt{(3-x)^2 + (2-y)^2 + 1^2}$ at $\sqrt{1^2} = 1$



Panel 5



$P(3,2,1)$ $\text{dist}(P, x\text{-axis}) = \sqrt{2^2+1^2}$

Panel 6

3D Objects

$P(x,y,z) \Rightarrow d = \sqrt{x^2+y^2+z^2} \Rightarrow d^2 = x^2+y^2+z^2$ *

All points that are a fixed distance to origin form a sphere

Def. $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = r^2$ is a sphere centered at (x_0, y_0, z_0) with radius r!

Ex. $x^2+y^2+z^2-2x-4y+8z+17=0$

$x^2-2x+1 + y^2-4y+4 + z^2+8z+16 = -17+1+4+16$

$(x-1)^2 + (y-2)^2 + (z+4)^2 = 4$ center $(1,2,-4)$ radius 2

Panel 7

Def: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$ is disk centered at $P(x_0,y_0,z_0)$ with radius r.

Ex: Find the center + radius of the sphere

$x^2+y^2+z^2+10x+4y+2z-19=0$

$x^2+10x+y^2+4y+z^2+2z=19$

$(x+5)^2+(y+2)^2+(z+1)^2=19+25+4+1$

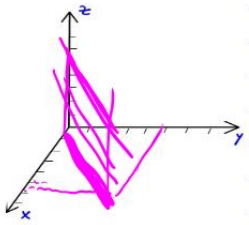
$(x+5)^2+(y+2)^2+(z+1)^2=49$

$r=7$ center $=(-5,-2,-1)$

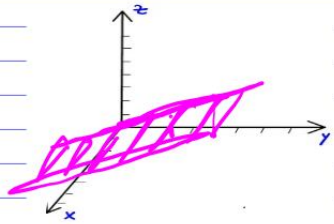
Panel 8

3D Objects

a) $y=x$



b) $z=\frac{1}{2}y$



Panel 9

Drawing 3D objects with Maple

Maple can easily draw 3D objects

$z = y^2$

Start Maple.

```
> with(plots);
> implicitplot3d(z = y^2, x = -3..3, y = -3..3, z = -1..9);
> plot3d(x^2, x = -3..3, y = -3..3);
> implicitplot3d(z^2 + y^2 = 4, x = -3..3, y = -3..3, z = -3..3);
```

$x^2 + y^2 = 1$

if $z =$ ____ then can use plot3d else use implicitplot3d

Panel 10

Ex: Use Maple to graph the following:

$x^2 + y^2 + z^2 = 4$

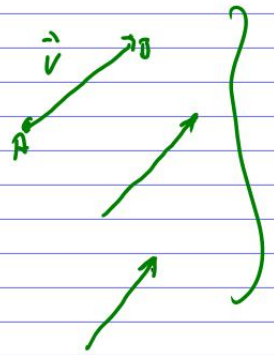
$y^2 + z^2 = 2$

$z = \sin(x) \cdot \cos(y)$ ✓

Panel 11

We understand points in 3D (and 2D).
Want to investigate more general objects: Vectors

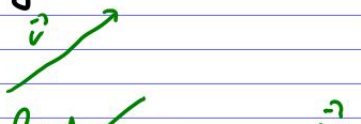
Def: A vector is a directed line segment, i.e. a part of a line with a length and a direction.
$\vec{v} = \vec{AB}$ is the vector from point A to point B.



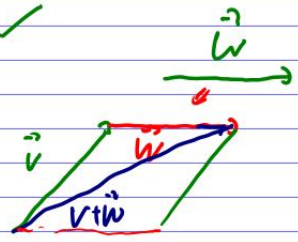
are all the same

Panel 12

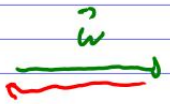
Vector Math, geometrically:
If $\vec{v}$ is a vector, then
$k \cdot \vec{v}$ is same dir., k-times as long

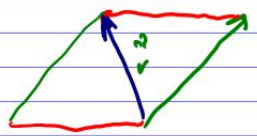


If $\vec{v}, \vec{w}$ are vectors, then
$\vec{v} + \vec{w}$ is diagonal of parallelogram $\vec{v}, \vec{w}$



If $\vec{v}, \vec{w}$ are vectors, then
$\vec{v} - \vec{w}$ is the other diagonal of parallelogram $\vec{v}, \vec{w}$





Panel 13

Vector Math, Algebraically vs. Geometrically

Algebraically v is described by components:

$\vec{v} = \langle v_1, v_2 \rangle$ or $\vec{v} = \langle v_1, v_2, v_3 \rangle$

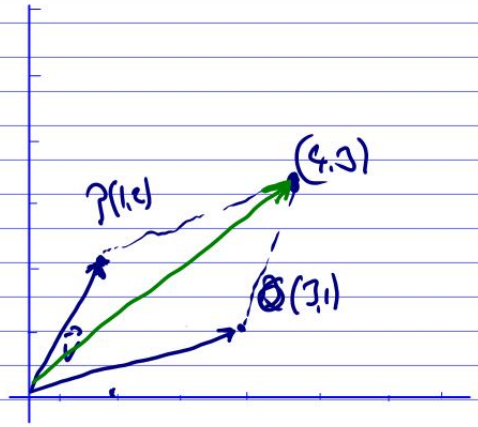
Ex: Suppose $\vec{v} = \langle 1,2 \rangle$, $\vec{w} = \langle 3,1 \rangle$. Find

$\vec{v} + \vec{w}$ $\langle 1,2 \rangle + \langle 3,1 \rangle$

$\langle 4,3 \rangle$

$\vec{v} + 2\vec{w}$ HW

$3\vec{v} - \vec{w}$ HW



Panel 14

Vectors: Some Definitions

Def: The Length or norm of a vector $\vec{v} = (v_1, v_2)$ is:

$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$ or $\sqrt{v_1^2 + v_2^2 + v_3^2}$ in 3D

Def: A unit vector $\vec{u}$ is a vector such that $\|\vec{u}\| = 1$

Note: If $\vec{v} = (v_1, v_2)$ is any non-zero vector, then $\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v}$ is a unit vector pointing in the same direction as $\vec{v}$.

Panel 15

Ex: $\left\langle \frac{1}{2}, \frac{3}{4} \right\rangle$ (a) $\left\| \left\langle \frac{1}{2}, \frac{3}{4} \right\rangle \right\| = \sqrt{\frac{1}{4} + \frac{9}{4}}$ Who is a unit vector?

$\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ (b) ✓ $= \sqrt{\frac{10}{4}} \neq 1$

Ex: Find unit vector in the direction of $\vec{v} = \langle 1, -5 \rangle$ and $\vec{w} = \langle 3, 2, -1 \rangle$