

Math 2411/2511 - Maple Optional Assignment

By now you are hopefully somewhat familiar with Maple. In these exercises you need to use Maple, perhaps together with some pencil and paper work, to answer some more involved questions. It is an optional assignment; I will only count it if it improves your final grade. Complete the work and paste (or type) your answers into this document.

We already saw the “plot3d” command to plot surfaces, i.e. graphs of functions $z = f(x, y)$. But Maple can also draw several surfaces in one plot. For example, if

$$f(x, y) = x^2 + y^2 \text{ and } l(x, y) = 2(x - 1) + 2(y - 1) + 2$$

then we can draw both surfaces via the Maple command:

> `plot3d({x^2 + y^2, 2(x - 1) + 2(y - 1) + 2}, x=-3..3, y=-3..3)`

Note that these functions are not accidentally chosen, but the second one is the tangent plane to the function at the point $P(1,1,2)$ – confirm that (finding tangent planes is in your notes).

1. Graph the surface and its tangent plane at the given point, choosing a domain and viewpoint (by rotating the graph) so that you get a good view of both surfaces:
 - a. $z = x^2 + xy + 3y^2$ at $P(1,1,5)$
 - b. $z = \arctan(xy^2)$ at $P(1,1, \pi/4)$

Maple can plot surfaces as well as contours. For example, to draw a contour plot for $f(x, y) = \sin(xy)$ we would use the command:

> `with(plots)`
> `contourplot(sin(x) cos(y), x=-2 pi..2 pi, y=-2 pi..2 pi)`

Compare that with a standard 3D plot of the surface.

2. Draw contour maps for each of the functions

$$f(x, y) = x^2 + y^2, g(x, y) = (x^2 + y^2)^{1/2}, h(x, y) = (4 - x^2 - y^2)^{1/2}$$

Compare the contour plots with the actual surfaces. What is the names for these surfaces?

Maple can differentiate easily using the *diff* command. Or even simpler, enter an expression, press enter, then right-click on the blue expression to pick the “Differentiate” command (note that you should

only enter the *right side* of the function definition). Use this “right-click” approach to compute the given partial derivatives:

3. $f(x, y) = \frac{xy \sin(x-y)}{1+x^2+y^2}$ - compute $f_x, f_y, f_{xx}, f_{xy}, f_{yx}, f_{yy}$ and confirm that $f_{xy} = f_{yx}$

Maple is very good at integration and can of course do multiple integrals just fine. It *cannot*, however, compute more specialized integrals such as a work integral without you first translating the question into an appropriate format. For example, to compute $\int_C xy^2 dx + \cos(y) dy$ along the curve C given by $r(t) = \langle 1 - t^2, 3t + 1 \rangle$ as $t = 0$ to $t = 2$ we need to transform that integral “by hand” into the integral:

$$\int_0^2 (1 - t^2)(3t + 1)^2(-2t)dt + \cos(3t + 1) 3dt$$

before Maple can evaluate it (the answer is $\sin(7) - \sin(1) + \frac{844}{5}$, incidentally)

4. Consider the work integral $\oint_C (xy^2)dx + (x^2y + \cos(x))dy$ where C is the closed curve consisting of the curve from (0,0) to (2,4) along $y = x^2$ and back from (2,4) to (0,0) along $y = 2x$. Use Maple to evaluate the integral first by going up the first curve and down the second one, and then by using Green’s theorem to convert the integral into a double integral over the region enclosed by the two curves. Confirm that both answers are the same.