Math 2511 – Calc 3 Practice Exam 1

This is a practice exam. The actual exam consists of questions of the type found in this practice exam, but will be shorter. If you have questions do not hesitate to send me email.

1. **Definitions**: Please state in your own words the meaning of the following terms:
   a) Vector
   b) Dot product, Cross product
   c) Angle between two vectors
   d) Unit vector
   e) Projection of **v** onto **w**
   f) Plane, Line, Space curve
   g) Distance between plane and point
   h) Intersection between a line and a point
   i) Smooth curve
   j) Tangent vector to a curve
   k) Unit tangent vector to a curve
   l) Normal vector to a curve
   m) Binormal vector
   n) Curvature
   o) Length of a curve

2. **True/False** questions:
   a) $u \cdot u = \|u\|^2$
   b) $<1,3,2>$ and $<-4,-2,5>$ are perpendicular
   c) $<1,3,-2>$ and $<2,6,4>$ are parallel
   d) P(1,2,3) is on the plane 3x – 2y – z = -2
   e) P(1,2,3) is on the line $r(t) =< 1 + 2t, 2 + 4t, 1 - t >$
   f) $v \times w = -w \times v$
   g) $v \cdot w = \|v\| \, \|w\| \sin(\theta)$
   h) $\frac{d}{dt}\|r(t)\| = \left\|\frac{d}{dt} r(t)\right\|$
   i) $\frac{d}{dt} p(t) \times r(t) = p'(t) \times r'(t)$
   j) $r(t) = \left\langle \sqrt{t} + 2, 3 - \sqrt[3]{t}, \sqrt[4]{t} \right\rangle$ is the equation of a line
   k) If $\|r(t)\| \equiv 1$ then $r(t) \times r'(t) = 0$
   l) The planes $x + 3y + 2z = 5$ and $4x + 2y - 5z = 0$ are perpendicular
   m) The distance between $x - y + z = 2$ and $x + y + z = 1$ is zero

3. **Vectors**: Suppose $u =< 7,-2,3 >$, $v =< -1,4,5 >$, and $w =< -2,1,-3 >$
   a) Are *u* and *v* orthogonal, parallel, or neither?
   b) Find graphically and algebraically $2u + 3v$ and $u - v$
   c) Find the angle between *v* and *w*
   d) Find $u \cdot v$ (dot product), $u \times v$ (cross product), $u \cdot (v \times w)$, and $\|u\|$
   e) Find the projection of *w* onto *u* and the projection of *u* onto *w*

4. **Lines and Planes**
   a) Find the equation of the plane spanned by $<1,3,-2>$ and $<2,1,2>$ through the point $P(1,2,3)$
   b) Find the equation of the plane through $P(1,2,3)$, $Q(1,-1,1)$, and $R(3,2,1)$
   c) Find the equation of the plane parallel to $x-y+z=2$ through $P(0,2,0)$
   d) Find the equation of a plane parallel to the lines $l_1(t) =< 1-2t, 2t, 3-t >$ and $l_2(t) =< t, 1-2t, 2+2t >$ through the point P(1, 0, 0)
   e) Find the equation of the line through $P(1,2,3)$ and $Q(1,-1,1)$
   f) Find the line parallel to the line $l(t) =< 1-2t, 2t, 3-t >$ through P(1,1,1)

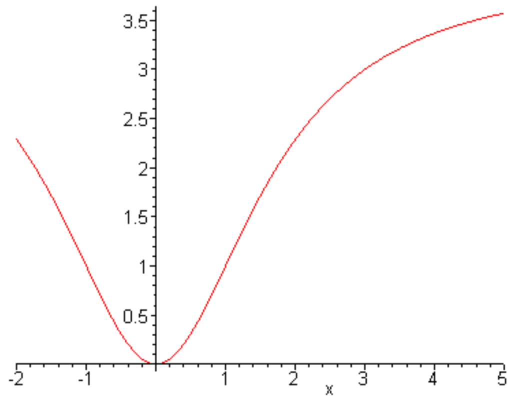
5. **Distances**
   a) Find the distance between the points P(1,2,3) and Q(1,2,5)
   b) Find the distance between the line $x-y=2$ and $P(1,2)$
   c) Find the distance between the line $l(t) =< 1-2t, 2t, 3-t >$ and the point $P(1, 2, 3)$
   d) Find the distance between the plane $x + y + z = 1$ and the point $P(1, 2, 3)$
   e) Find the distance between the plane $x + 2y - z = 1$ and the line $l(t) = < 4, -1, 2 >$
   f) Find the distance between the planes $x-y+z=2$ and $2x-2y+2z=5$

6. **Intersections**
   a) Are the following lines parallel, skew, or intersecting? If they intersect, find the point of intersection: $l_1(t) =< 2,-1,2 > +t < 3,1,1 >$ and $l_2(t) =< -1,0,0 > +t < 3,-1,2 >$
   b) Do the plane $x-y+z=2$ and the line $l(t) =< 1+t, 2t, 1-5t >$ intersect? If so, where?
   c) Do the planes $x - y + z = 0$ and $2x - z = 3$ intersect? If so, find the line of intersection.

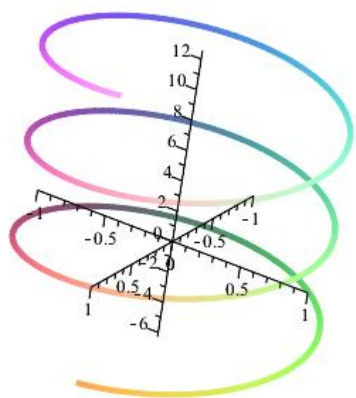
7. **Vector valued functions**:
   a) Find $r'(t)$ if $r(t) =< 6t, -7t^2, t^3 >$
   b) Find $r''(t)$ if $r(t) =< a\cos^3(t), a\sin^3(t), t\sin(t) >$
   c) If $r(t) =< 4t, t^2, t^3 >$, find $r'(t)$, $r''(t)$, $\frac{d}{dt}\|r(t)\|$, $\left\|\frac{d}{dt}r(t)\right\|$
   d) If $r(t) =< e^t, 3t^3, \frac{3}{6t} >$ some curve, find $\int_1^2 r(t)dt$ and $\int_1^2 \|r'(t)\| \, dt$
   e) If $r(t) =< t, \frac{1}{t} >$, find $T(t)$, $N(t)$
   f) Repeat (e) for $r(t) =< e^t\cos(t), e^t\sin(t) >$ for $t = \frac{\pi}{2}$ (Hint: there is a neat trick to find N(t), which only works in 2 dimensions)
   g) Repeat (e) for $r(t) =< 2t, 1, t^3 >$ and also find the binormal at t = 1.
   h) If $r(t) =< 3-3t, 4t >$, find the arc length of the curve between 0 and 1
   i) If $r(t) =< 4t, 3\cos(t), 3\sin(t) >$, find the arc length of the curve between 0 and $\frac{\pi}{2}$
   j) If $r(t) =< 2t^2, 3t-1, \cos(t) >$, find the arc length of the curve between 0 and π (Hint: if the integration becomes tricky, try Wolfram Alpha)

k) Find the curvature of $r(t) = < t, 3t^2, \frac{t^2}{2} >$

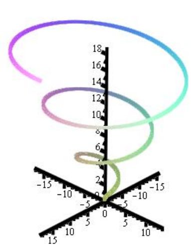
8. **Picture:** Sketch the circle that fits the graph below the best at the points (0,0) and *(3,3)*. At which of the two points is the curvature smaller? Also sketch the unit tangent and unit normal for *(3,3)*.

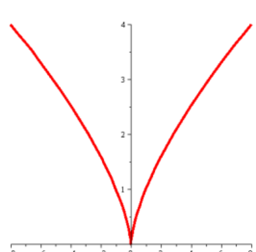


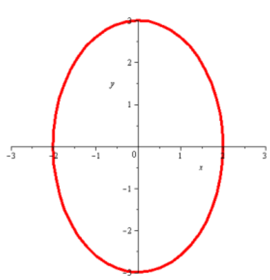
9. **Picture:** Sketch the unit tangent, normal, and binormal to the curve <cos(t), sin(t), t> as best as possible, when time *t =0*. A sketch suffices, you don't have to compute the actual vectors:

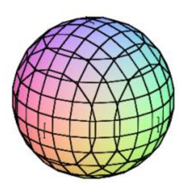


10. **Picture**: Match the following functions to their corresponding plots.









$r(t) = < t^3, t^2 >$ $\quad$ $x^2 + y^2 + z^2 = 1$ $\quad$ $r(t) = < t\cos(t), t\sin(t), t >$ $\quad$ $r(t) = < 2\sin(t), 3\cos(t) >$

11. **Story problem:** Coming up soon …

12. **Prove** the following facts:
    a) Show that $u \times v = -(v \times u)$
    b) Show that $u \cdot (v \times u) = 0$
    c) Show that if $y = f(x)$ is a function that is twice continuously differentiable, then the curvature of f at a point x is $K = \dfrac{|f''(x)|}{\left(1+[f'(x)]^2\right)^{3/2}}$
    d) Prove that the curvature of a line in space is zero

13. Here are a few **miscellaneous** questions:
    a) In the definition of unit tangent to a curve r(t) we specify that the curve must be smooth. Why?
    b) If two lines in three dimensional space are not parallel, do they have to intersect?
    c) The curvature of a parabola is largest at its vertex, as we mentioned in class. Where, do you think, is the curvature of a simple 3rd degree polynomial $y = x^3$ the largest?

    Some more questions coming up later …