Calc 3 - Assignment (Last One - yeah)

Note Title 11/17/2011

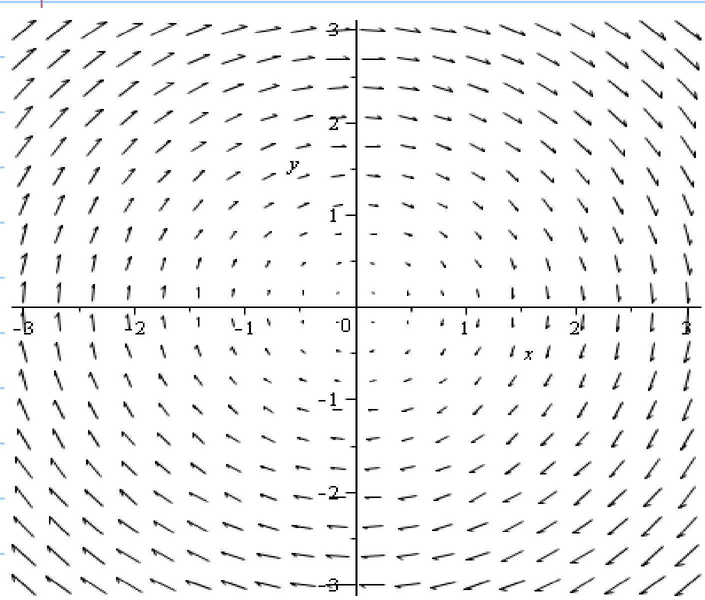
1. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ for the given curve $r(t)$.

a) $\vec{F} = \langle xy, 3y^2 \rangle$, $r(t) = \langle 11t^4, t^3 \rangle$, $t \in [0,1]$

b) $\vec{F} = \langle x+y, y-z, z^2 \rangle$, $r(t) = \langle t^2, t^3, t^2 \rangle$, $t \in [0,1]$

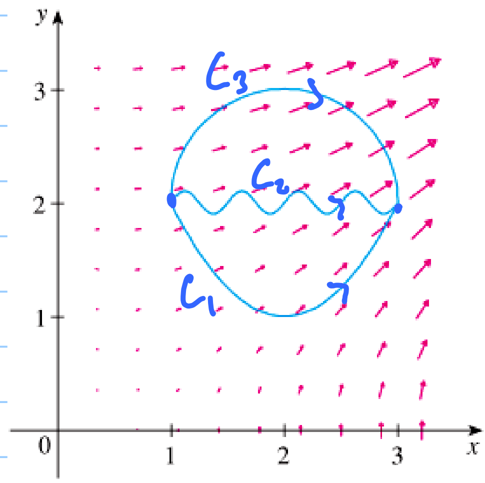
c) $\vec{F} = \langle z, y, -x \rangle$, $r(t) = \langle t, \sin(t), \cos(t) \rangle$, $t \in [0, \pi]$

2. Let $\vec{F}$ be the vector field shown in the figure below. Let C_1 be the line segment from $(-3,-3)$ to $(-3,3)$, and C_2 a circle with radius 3 and center at the origin. Are $\int_{C_1} \vec{F} \cdot d\vec{r}$ and $\int_{C_2} \vec{F} \cdot d\vec{r}$ positive, negative, or zero?



The vector field $\langle y, -x \rangle$

(3) The figure below shows the vector field $\vec{F} = \langle 2xy, x^2 \rangle$ and three curves from $(1,2)$ to $(3,2)$. Explain why $\int_C \vec{F}\cdot d\vec{r}$ has the same value for all three curves C_1, C_2, C_3, and find that value.

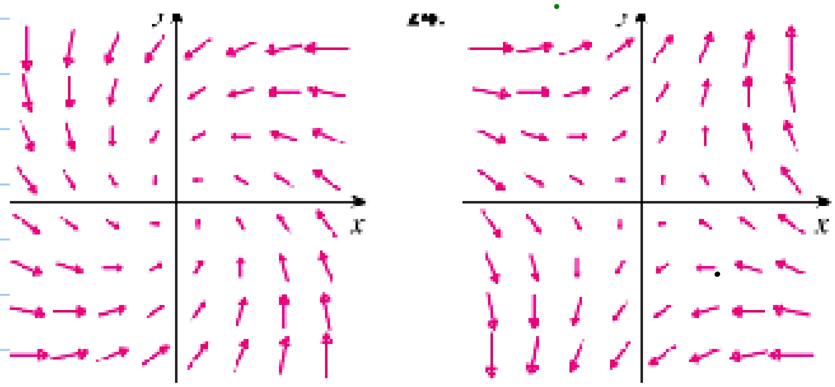


(4) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = \langle x^2, y^2 \rangle$ and C is the part of the parabola $y = 2x^2$ from $(-1,2)$ to $(2,8)$ using (a) line integration, and (b) the Fund. Thm. They should agree.

(5) Evaluate $\int_C yz\,dx + xz\,dy + (xy + 2z)\,dz$, where C is the line segment from $(1,0,-2)$ to $(4,6,3)$ using (a) line integration, and (b) the Fund. Thm.

(6) Find $\int_C \tan(y)\,dx + x\sec^2(y)\,dy$, C any path from $(1,0)$ to $(2, \pi/4)$

(7) Which vector field is conservative?



(8) Is $\int_C y\,dx + x\,dy + xyz\,dz$ independent of the path?

(9) Evaluate $\int_C 2(x+y)dx + 2(x+y)dy$, C curve from (-2, 2) to (4, 3)

(10) Find the work done by the force field $F = \langle 9x^2y^2, 6x^3y - 1 \rangle$ from P(0,0) to Q(5,9)

(11) Find $\oint_C y\sin(x)\,dx - \cos(x)\,dy$

where C bounds the region between $y=0$ and $y = 4 - x^2$

Find a conservative vector field that has the given potential:

$$f(z,y,z) = \sin(x^2 + y^2 + z^2)$$

Find $div(\nabla \cdot F)$ and $curl(F) = \nabla \times F$

$$F(x,y,z) = < x^2 z, y^2 x, y + 2z >$$

Evaluate $\int_C (x-y)dx + xdy$ if C is the graph of $y^2 = x$ from (4,-2) to (4,2)

Find the work done by F(x,y,z) along the curve $< t, t^2, t^3 >$ from (0, 0, 0) to (2, 4, 8), where $F(x,y,z) = < y, z, x >$

Check which of the following vector fields is not conservative.

$$F(x,y) = < 3x^2 y + 2, x^3 + 4y^3 >$$

$$F(x,y) = < e^x, 3 - e^x \sin(y) >$$

$$F(x,y,z) = < 8xz, 1 - 6yz^2, 4x^2 - 9y^2 z^z >$$

Show that the line integrals are independent of the path, and find their value:

$$\int_{(-1,2)}^{(3,11)} (y^2 + 2xy)dx + (x^2 + 2xy)dx$$

$$\int_{(1,0,2)}^{(-2,1,3)} (6xy^3 + 2z^2)dx + (9x^2 y^2)dy + (4xz + 1)dz$$