Panel 1

$\oint_C (e^y - x^2 y)dx + (xy^2 + xe^y)dy$, C = unit circle

$\int_C \vec{F}\, d\vec{r}$ closed curve!

Green

Long way

$$= \iint \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA = \iint y^2 + \cancel{e^y} - (\cancel{e^y} - x^2)\, dA$$

$$= \iint_{Disk} x^2 + y^2\, dA = \int_0^{2\pi}\int_0^1 r^2\, r\, dr\, d\theta$$

$$= \frac{1}{4} 2\pi = \frac{\pi}{2}$$

$x = r\cos\theta$

$y = r\sin\theta$

$dA = r\,dr\,d\theta$

1

Panel 2

$\int \langle y, z, x\rangle\, d\vec{r}$, circle in yz plane at $x = 3$

curl(F)

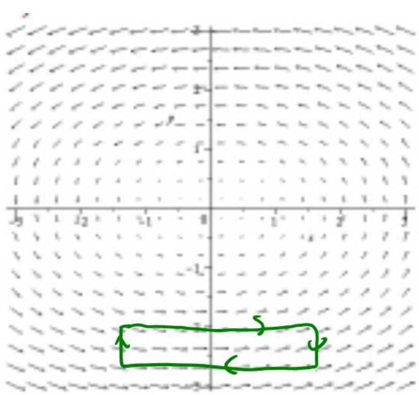
$r(t) = \langle 3, \cos(t), \sin(t)\rangle$

$$\int y\,dx + z\,dy + x\,dz = \int_0^{2\pi} \cos(t)\cdot 0 - \sin^2(t)\,dt + 3\cos(t)\,dt$$

= Maple!

2

Panel 3



3

Panel 4

$F = \langle x^3 y^2 z, x^2 z, x^2 y\rangle$

$$\operatorname{div}(F) = \frac{\partial}{\partial x} x^3 y^2 z + \frac{\partial}{\partial y} x^2 z + \frac{\partial}{\partial z} x^2 y =$$

$$3x^2 y^2 z + 0 + 0$$

$$\begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ x^3 y^2 z & x^2 z & x^2 y \end{vmatrix} = \langle x^2 - x^2, (2xy - x^3 y^2), 2xz - 2x^3 yz\rangle$$

4

Panel 5

$$F = \langle 6xy^2 - 3x^2, \underline{6x^2y + 3y^2 - 7} \rangle$$

$$f_x = 6xy^2 - 3x^2 \Rightarrow f = 3x^2y^2 - x^3 + C(y)$$

$$f_y = \cancel{6x^2y} + C'(y) = \cancel{6x^2y} + 3y^2 - 7 \Rightarrow C(y) = y^3 - 7y + c$$

$$\underline{f = 3x^2y^2 - x^3 + y^3 - 7y + c}$$

Panel 6

$$F = \langle 4xy + z^2, 2x^2 + 6yz, 2xz \rangle$$

$$f_x = 4xy + z^2 \Rightarrow f = 2x^2y + xz^2 + C(y,z)$$

$$f_y = \cancel{2x^2} + C_y(y,z) = \cancel{2x^2} + 6yz \Rightarrow C(y,z) = 3y^2z + C(z)$$

$$f = 2x^2y + xz^2 + 3y^2z + C(z)$$

$$f_z = \cancel{2xz} + 3y^2 + C'(z) = \cancel{2xz}$$ No good!

$$f = 2x^2y + xz^2 + xy^3z + zy$$

$$\langle 4xy + z^2 + y^3, 2x^2 + 3xy^2 + z, 2xz + y \rangle$$

Panel 7

$$\langle 4xy + z^2 + y^3, 2x^2 + 3xy^2 + z, 2xz + y \rangle$$

$$f_x = 4xy + z^2 + y^3 \Rightarrow f = \underline{2x^2y + xz^2 + xy^3} + C(y,z)$$

$$f_y = \cancel{2x^2} + \cancel{3xy^2} + C_y(y,z) = \cancel{2x^2} + \cancel{3xy^2} + z$$

$$\Rightarrow C(y,z) = yz + C(z)$$

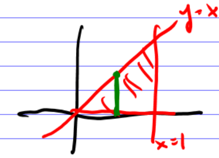
$$f = 2x^2y + xz^2 + xy^3 + yz + C(z)$$

$$f_z = \cancel{2xz} + \cancel{y} + C'(z) = \cancel{2xz} + \cancel{y} \Rightarrow C(z) = c$$

$$\underline{f = 2x^2y + xz^2 + xy^3 + yz + c}$$

Panel 8

$$\iint \cos(x^2)\,dA = \cancel{\iint \cos(x^2)\,dx\,dy} = \int_0^1\int_0^x \cos(x^2)\,\underline{dy}\,dx$$



$$\int ds = \quad r(t) = \langle t^2, 1+t \rangle, \; t \in [0,2]$$

$$\int_0^2 \sqrt{(x')^2 + (y')^2}\,dt = \int_0^2 \sqrt{4t^2+1}\,dt = \underline{\text{Maple!}}$$

Panel 9

$\int x^2 y^3\, dx$, $r(t) = \langle t^2, t^3 \rangle$, $t \in [0,2]$

$\int_0^2 (t^2)^2 (t^3)^3\, 2t\, dt$

$\int x^2 - y + 3z\, ds$, $r(t) = \langle t, 2t, 3t \rangle$, $t \in [0,1]$

$\int_0^1 \left(t^2 - (2t) + 3(3t)\right) \sqrt{1+4+9}\, dt = \sqrt{14} \int_0^1 t^2 - 2t - 9t\, dt$

$\int \langle y, x^2 \rangle \cdot dr$, $r = \langle 4-t, 4t-t^2 \rangle$

$\int y\, dx + \int x^2\, dy = \int (4t-t^2)(-1)\, dt + (4-t)^2 (4-2t)\, dt$

9

Panel 10

$\int_C 2xyz\, dx + x^2 z\, dy + x^2 y\, dz =$ from $(0,0,0)$ to $(1,4,3)$

$= f(1,4,3) - f(0,0,0) = 4 \cdot 3 - 0 = 12$

$\langle 2xyz, x^2 z, x^2 y \rangle$

$f_x = 2xyz \Rightarrow f = x^2 y z + C(y,z)$

$f_y = \cancel{x^2 z} + C_y = \cancel{x^2 z} \Rightarrow C_y = 0 \Rightarrow C(z)$

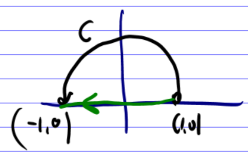
$f_z = x^2 y + C'(z) = x^2 y \Rightarrow C = c$

$f = x^2 y z + c$

10

Panel 11

$\int_C \langle y^3 + 1, 3xy^2 + 1 \rangle \cdot d\vec{r}$



① $r(t) = \langle \cos(t), \sin(t) \rangle$, $t \in [0, \pi]$ too much

② ~~Green thm~~ C is not closed

③ Conservative ✓ ⇒ find potential f ; $f(-1,0) - f(1,0)$

Use other curve: $r(t) = \langle t, 0 \rangle$, t from 1 to -1

$\int y^3 + 1\, dx + (3xy^2 + 1)\, dy = \int_1^{-1} 0 + 1\, dt + (\quad) \cdot 0 = -2$

11

Panel 12

$\int y^3 x\, dx + 3xy^2\, dy$ line from $(-1,0)$ to $(2,3)$

$r(t) = \langle -1, 0 \rangle + t \langle 3, 3 \rangle$, $t \in [0,1]$

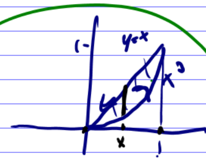
$= \langle -1 + 3t, 3t \rangle$

$\int_0^1 (3t)^3 (-1+3t)\, 3\, dt + 3(-1+3t)(3t)^2\, 3\, dt$

12

Panel 13

$$\int_C \overset{M}{y^3}\,dx + \overset{N}{(x^3+3xy^2)}\,dy$$



$$\iint_R \left(3x^2 + \cancel{3y^2} - \cancel{3y^2}\right) dA$$

$$\int_0^1 \int_{x^3}^{x} 3x^2\,dy\,dx$$

$$\iint_{Disk} 3x^2\,dA = \int_0^{2\pi}\int_0^3 3r^2\cos^2(\theta)\,r\,dr\,d\theta$$

$x = r\cos\theta$

$y = r\sin\theta$

13

Panel 14

Thm: $\iint_R dA = \text{area}(R)$

$\frac{1}{2}\oint_C x\,dy - y\,dx = \text{area}(R)$, R = inside of closed curve C.

Look at lectures!

Skipped: ~~Lagrange Multipliers~~

Chain Rule (important theory)

Stokes Thm

(Gauss Thm)

14