Panel 1

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$D_{\vec{u}}(f) = \lim_{h \to 0} \frac{f(x+hu_1, y+hu_2) - f(x, y)}{h}$$

$$\stackrel{\text{thm}}{=} (\nabla f) \cdot (u), \quad \|u\| = 1$$

$\int f \, dx$ area under f, $f \geq 0$

$\int\int f(x,y)\, dA$ volume under $z = f(x,y)$, $f \geq 0$

$$\int_a^b \int_c^d f(x) g(y) \, dy \, dx = \int_a^b f(x) \int_c^d g(y) \, dy \, dx = \left(\int g(y) dy\right)\left(\int f(x) dx\right)$$

1

Panel 2

$$\lim_{(x,y) \to (0,0)} \frac{xy}{x^2 + y^2}$$

$$\lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x - y} = \lim_{(x,y) \to 0} \frac{(x+y)\cancel{(x-y)}}{\cancel{(x-y)}} = 0$$

(1) Subst. + Hope!

(1.5) Factor if possible

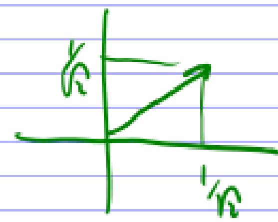
(2) $x \to 0, y = 0$; 0
$y \to 0, x = 0$
$x = y \to 0$ $\frac{1}{2}$
$x^2, y \to 0$
$y^2, x \to 0$

(4) Prove or avoid

2

Panel 3

$f(x,y) = xye^{xy}$ at $(-2,0)$, $\theta = \pi/4$



$D_{\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle}(f)\Big|_{(-2,0)}$

$\langle f_x, f_y \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle \Big|_{(-2,0)}$

$\langle ye^{xy} + xy^2e^{xy}, xe^{xy} + x^2ye^{xy} \rangle \cdot \langle 1,1 \rangle \cdot \frac{1}{\sqrt{2}} \Big|_{(-2,0)}$

$\left(\cancel{ye^{xy}} + \cancel{xy^2e^{xy}} + xe^{xy} + \cancel{x^2ye^{xy}}\right)\frac{1}{\sqrt{2}} \Big|_{(-2,0)}$

$-2e^0 \cdot \frac{1}{\sqrt{2}} = \underline{-2/\sqrt{2}}$

3

Panel 4

$f(x,y) = 4xy - x^4 - y^4$

① $f_x = \cancel{4}y - \cancel{4}x^3 = 0 \Rightarrow \underline{y = x^3}$

$f_y = \cancel{4}x - \cancel{4}y^3 = 0$

$x - x^9 = 0$

$x(1-x^8) = 0$

$(y=0, \underline{x=0})$ $(1,1)$ $(-1,-1)$

$\boxed{②}$ $H = \begin{pmatrix} -12x^2 & 4 \\ 4 & -12y^2 \end{pmatrix}$ $D = 144x^2y^2 - 16$

$(0,0)$: $D < 0 \to$ saddle

$(1,1)$: $D > 0$, $f_{xx} < 0$ $\underline{\text{max}}$

$(-1,-1)$: $D > 0$, $f_{xx} < 0$ $\underline{\text{max}}$

4

Panel 5

a) $\int_0^1\int_0^2 xy^2\,dx\,dy$

b) $\cancel{\int_0^\pi\int_0^{\pi/2}\sin(x)\cos(y)\,dy\,dx}$

c) $\int_0^2\int_{x^2}^{x}(x^2+2y)\,dy\,dx$

d) $\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\sqrt{x^2+y^2}\,dy\,dx$

e) $\cancel{\int_0^a\int_0^b\int_0^c x^2+y^2+z^2\,dx\,dy\,dz}$

f) $\int_0^{\sqrt{\pi}}\int_y^1\cos(x^2)\,dx\,dy$

$$\int_0^1\int_0^2 x y^2\,dx\,dy = \int_0^1 \frac{1}{2}x^2y^2\Big|_{x=0}^{x=2}\,dy =$$

$$= \int_0^1 2y^2 - 0\,dy =$$

$$= \int_0^1 2y^2\,dy = \frac{2}{3}y^3\Big|_0^1 = \frac{2}{3}$$

$$\int_0^2\int_{x^2}^{x} x^2+2y\,dy\,dx = \int_0^2 x^2y+y^2\Big|_{y=x^2}^{y=x}\,dx =$$

$$= \int_0^2 (x^3+x^2)-(x^4+x^4)\,dx = \int_0^2 -2x^4+x^3+x^2\,dx =$$

5

Panel 6

a) $\int_0^1\int_0^2 xy^2\,dx\,dy$

b) $\int_0^\pi\int_0^{\pi/2}\sin(x)\cos(y)\,dy\,dx$

c) $\int_0^2\int_{x^2}^{x}(x^2+2y)\,dy\,dx$

d) $\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\sqrt{x^2+y^2}\,dy\,dx$

e) $\int_0^a\int_0^b\int_0^c x^2+y^2+z^2\,dx\,dy\,dz$

f) $\int_0^{\sqrt{\pi}}\int_y^1\cos(x^2)\,dx\,dy$

$x = r\cos\theta$

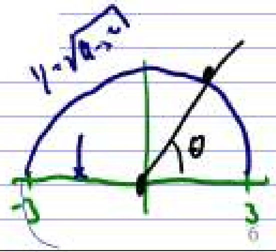
$y = r\sin\theta$

$dx\,dy = \boxed{r}\,dr\,d\theta$

$$= \int_0^\pi\int_0^3 \sqrt{r^2\cos^2\theta + r^2\sin^2\theta}\;r\,dr\,d\theta =$$

$$= \int_0^\pi\int_0^3 r^2\,dr\,d\theta = \int_0^\pi \frac{1}{3}r^3\Big|_0^3\,d\theta =$$

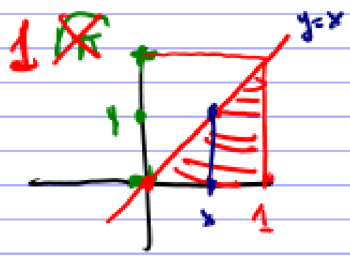
9π



6

Panel 7

$$\int_0^1\int_y^1 \cos(x^2)\,dx\,dy = \int_0^1\int_0^x \cos(x^2)\,dy\,dx = \int_0^1 y\cos(x^2)\Big|_{y=0}^{y=x}\,dx$$



x from $x=y$ to $x=1$

$$= \int_0^1 x\cos(x^2) - 0\,dx$$

$$= \int_0^1 x\cos(x^2)\,dx =$$

$$= \frac{1}{2}\sin(x^2)\Big|_{x=0}^{x=1} = \frac{1}{2}\sin(1)$$

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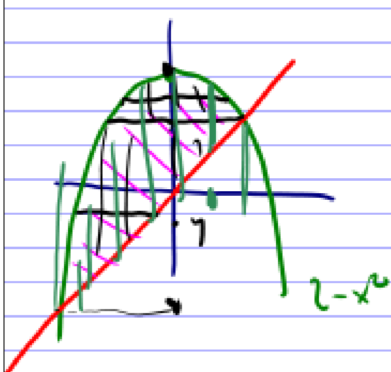
Panel 8

$y=2-x^2$, $y=x$, $y=0$



2 ints $\iint \quad dx\,dy$

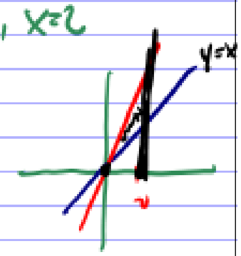
$\iint \quad dy\,dx$

8

Panel 9

$$\iint_R \frac{y}{x^2+y^2}\, dA$$

$y=x,\ y=2x,\ x=2$



$$\int_0^2 \int_x^{2x} \frac{y}{x^2+y^2}\, dy\, dx = \int \frac{1}{2} \ln(x^2+y^2)\Big|_{y=x}^{y=2x} dx$$

$$\int \ln(2x^2)\, dx =$$

$$\int \ln(2) + 2\ln(x)\, dx$$

$$\int 1 \cdot \ln(x)\, dx$$

int. parts

$$\iint \frac{y}{x^2+y^2}\, dx\, dy = \int y \left(\int \frac{1}{x^2+y^2}\, dx \right) dy$$

$\int \frac{1}{1+x^2}$ nicer $= \int y \int \frac{1}{y^2\left(\frac{x^2}{y^2}+1\right)}\, dx$ harder!

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