

Panel 1

1. Let $f(x, y) = 3xy^3 + 2x^2y$

a. Find f_x

$$f_x = 3y^3 + 4xy$$

b. Find $\frac{\partial^2 f}{\partial x \partial y}$

$$f_{xy} = 9y^2 + 4x \quad \checkmark$$

c. Compute ∇f

$$f_x, \quad f_y = 9xy^2 + 2x^2$$

$$f_{yx} = 9y^2 + 4x \quad \checkmark$$

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Panel 2

2. Let $g(x, y, z) = xy \tan(x^2y^3z^4)$. Compute ∇g

$$g_x = y \cdot \tan(x^2y^3z^4) + xy \cdot \sec^2(x^2y^3z^4) \cdot 2xg^3z^4$$

$$g_y = x \cdot \tan(x^2y^3z^4) + xy \cdot \sec^2(x^2y^3z^4) \cdot 3y^2x^2z^4$$

$$g_z = xy \sec^2(x^2y^3z^4) \cdot 4 \cdot x^2y^3z^3$$

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Panel 3

3. Consider $h(x, y, z, w) = 2xy - 3yz + 4zw - 5xw$. Compute h_{xyzw}

$$h_x = 2y - 5w$$

$$h_{xy} = 2$$

$$h_{xyz} = 0$$

$$h_{xyzw} = 0$$

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Panel 4

4. Let $f(x, y) = \frac{xy \sin(xy)}{\cos(xy)}$. Find f_x and f_y

$$f_x = \frac{(y \cdot \sin(xy) + xy \cos(xy) \cdot y) \cos(xy) - xy \sin(xy) \cdot (-\sin(xy) \cdot y)}{\cos^2(xy)}$$

$$f_y = \text{similar}$$

$$\frac{f'_y - f_y f'_x}{q^2}$$

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Panel 5

5. Consider $f(x, y) = \frac{x}{y}$. Find f_{xx} , f_{yy} , f_{xy} , and f_{yx} and confirm that $f_{xy} = f_{yx}$

$$f_x = \frac{1}{y} = y^{-1} \quad f_{xx} = 0$$

$$f_y = -x y^{-2} \quad f_{yy} = +2x y^{-3}$$

$$f_{xy} = -y^{-2}$$

$$f_{yx} = -y^{-2}$$

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Panel 6

6. Compute $\int (2xy^2 + 3x^2y) dy$

$$\int (2x^1 + 3x^2) dx$$

$$\int x^2 y^2 + x^3 y + C dy$$

$$\frac{1}{3} x^2 y^3 + \frac{1}{2} x^3 y^2 + C y + D$$

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Panel 7

7. Find $\int_1^2 \int_{\ln(2)}^{\ln(3)} x e^y (dy) dx$

$$\int_1^2 x e^y \Big|_{y=\ln(2)}^{y=\ln(3)} dx =$$

$$\int_1^2 3x - 2x dx = \int_1^2 x dx = \frac{1}{2} x^2 \Big|_1^2 = 2 - \frac{1}{2} = \frac{3}{2}$$

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Panel 8

8. Compute $\int_0^1 \int_0^2 \int_0^3 xy + yz + xz (dx) dy dz$

$$\int_0^1 \int_0^2 \left(\frac{1}{2} x^2 y + x y z + \frac{1}{2} x^2 z \right) \Big|_{x=0}^{x=3} dy dz =$$

$$\int_0^1 \int_0^2 \left(\frac{9}{2} y + 3 y z + \frac{9}{2} z \right) dy dz =$$

$$\int_0^1 \left(\frac{9}{2} y^2 + 3 y^2 z + \frac{9}{2} 6 y \right) \Big|_{y=0}^{y=2} dz =$$

$$\int_0^1 (9 + 6z + 9z) dz = \int_0^1 (9 + 15z) dz = 9z + \frac{15}{2} z^2 \Big|_0^1 = \underline{\underline{9 + \frac{15}{2}}}$$

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Panel 9

9. Find $\int_0^2 \int_1^3 6xy^2 dy dx$ and $\int_1^3 \int_0^2 6xy^2 dx dy$ ✓

$$\int_0^2 \int_1^3 6xy^2 dy dx = \int_0^2 6x \left[\frac{y^3}{3} \right]_{y=1}^{y=3} dx =$$

$$= \int_0^2 (54x - 2x) dx = \int_0^2 52x dx = \left[\frac{52x^2}{2} \right]_0^2 = \underline{\underline{104}}$$

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Panel 10

10. Evaluate $\int_e^{e^2} \int_0^1 \frac{x}{y} dx dy$

$$\int_e^{e^2} \left[\frac{1}{2} \frac{x^2}{y} \right]_{x=0}^{x=1} dy = \int_e^{e^2} \frac{1}{2y} dy = \frac{1}{2} \int_e^{e^2} \frac{1}{y} dy =$$

$$= \frac{1}{2} \ln(y) \Big|_e^{e^2} = \underline{\underline{1 - \frac{1}{2} = \frac{1}{2}}}$$

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Panel 11

11. Find $\int_0^1 \int_0^1 x \sin(xy) dx dy$ and $\int_0^1 \int_0^1 x \sin(xy) dy dx$. Which way, if any, is easier?

$$\int_0^1 -\cos(xy) \Big|_{y=0}^{y=1} dx = \int_0^1 -\cos(x) + 1 dx$$

requires Int. parts!

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Panel 12

Quit Time:

Relative max/min: $\nabla f = 0$

$$H = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} \quad D = f_{xx}f_{yy} - (f_{xy})^2$$

$D > 0, f_{xx} > 0$: min
 $D > 0, f_{xx} < 0$: max
 $D < 0$: saddle

Abs. max/min: $(x,y) \in [a,b] \times [c,d]$

$\nabla f = 0$

(a, c)	$\frac{\text{max}}{\text{or}} \frac{\text{min}}{\text{min}}$
(a, d)	
(b, c)	
(b, d)	

+ critical

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Panel 15

$f(x,y) = x^2 + y^2 + x^2y + 4$, $x \in [-1,1] \times [-1,1]$ outside
 $f_x = 2x + 2xy = 0$ $x(1+y) = 0$, $x=0$, $y=-1$, $y=1$
 $f_y = 2y + x^2 = 0$ $2y + x^2 = 0$, $-1 < x^2 < 0$, $x^2 = 2 = 0$
 ∇f

	(x,y)
$x = -1$: $f(y) = 1 + y^2 + y + 4 = y^2 + y + 5 \Rightarrow y = -1/2$	$(-1, -1)$
$x = 1$: $f(y) = y^2 + y + 5 \Rightarrow y = -1/2$	$(1, -1)$
$y = -1$: $f(x) = x^2 + 1 - x^2 + 4 = 5$	$(1, 1)$
$y = 1$: $f(x) = x^2 + x^2 + 5 = 2x^2 + 5 \Rightarrow x = 0$	$(0, 1)$
	$(0, 0)$
	$(-1, 0)$
	$(-1, -1/2)$
	$(0, 1)$

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Panel 16

Quiz Name: _____

① Find all rel. extrema of $f(x,y) = x^2 - y^2 + 4xy$ and classify them

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Panel 17

② Find all absolute extrema of $f(x,y) = 2x^2 + 4y^2 - 4x$
on $[0,2] \times [0,3]$

$$\nabla f = 0$$

$$\text{Set } x=0: f(y) =$$

$$\text{Set } x=2: f(y) =$$

$$\text{Set } y=0: f(x) =$$

$$\text{Set } y=3: f(x) =$$

(x,y)	$f(x,y)$
$(0,0)$	
$(0,3)$	
$(2,0)$	
$(2,3)$	