Panel 1

1. Let $f(x,y) = 3xy^3 + 2x^2y$

   a. Find f_x

   $f_x = 3y^3 + 4xy$

   b. Find $\frac{\partial^2 f}{\partial x \partial y}$

   $f_{xy} = 9y^2 + 4x$ ✓

   c. Compute ∇f

   f_x, $f_y = 9xy^2 + 2x^2$

   $f_{yx} = 9y^2 + 4x$ ✓

1

Panel 2

2. Let $g(x,y,z) = xy\tan(x^2y^3z^4)$. Compute ∇g

$g_x = g \cdot \tan(x^2y^3z^4) + xy \cdot \sec^2(x^2y^3z^4) \cdot 2xy^3z^4$

$g_y = x \cdot \tan(x^2y^3z^4) + xy \cdot \sec^2(x^2y^3z^4) \cdot 3y^2x^2z^4$

$g_z = xy \sec^2(x^2y^3z^4) \cdot 4 \cdot x^2y^3z^3$

2

Panel 3

3. Consider $h(x,y,z,w) = 2xy - 3yz + 4zw - 5xw$. Compute h_{xyzw}

$h_x = 2y - 5w$

$h_{xy} = 2$

$h_{xyz} = 0$

$h_{xyzw} = 0$

3

Panel 4

4. Let $f(x,y) = \frac{xy\sin(xy)}{\cos(xy)}$. Find f_x and f_y

$f_x = \frac{(y\cdot\sin(xy) + xy\cos(xy)\cdot y)\cos(xy) - xy\sin(xy)\cdot(-\sin(xy)\cdot y)}{\cos^2(xy)}$

f_y = similar

$\frac{f'g - fg'}{g^2}$

4

Panel 5

5. Consider $f(x,y) = \frac{x}{y}$. Find f_{xx}, f_{yy}, f_{xy}, and f_{yx} and confirm that $f_{xy} = f_{yx}$

xy^{-1}

$f_x = \frac{1}{y} = y^{-1}$ $f_{xx} = 0$

$f_y = -xy^{-2}$ $f_{yy} = +2xy^{-3}$

$f_{xy} = -y^{-2}$

$f_{yx} = -y^{-2}$

5

Panel 6

6. Compute $\int\int 2xy^1 + 3x^2y\,dx\,dy$

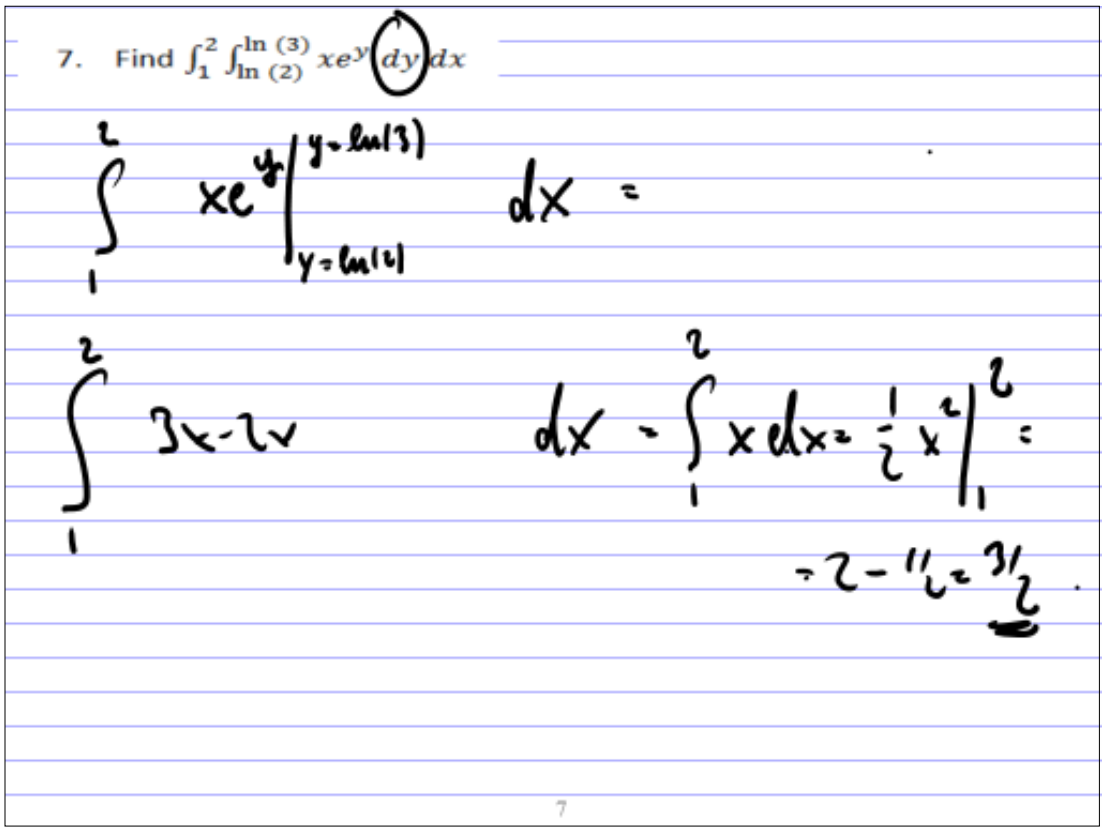
$\int 2x(1) + 3x^2\,dx$

$\int x^2y^2 + x^3y + C\,dy$

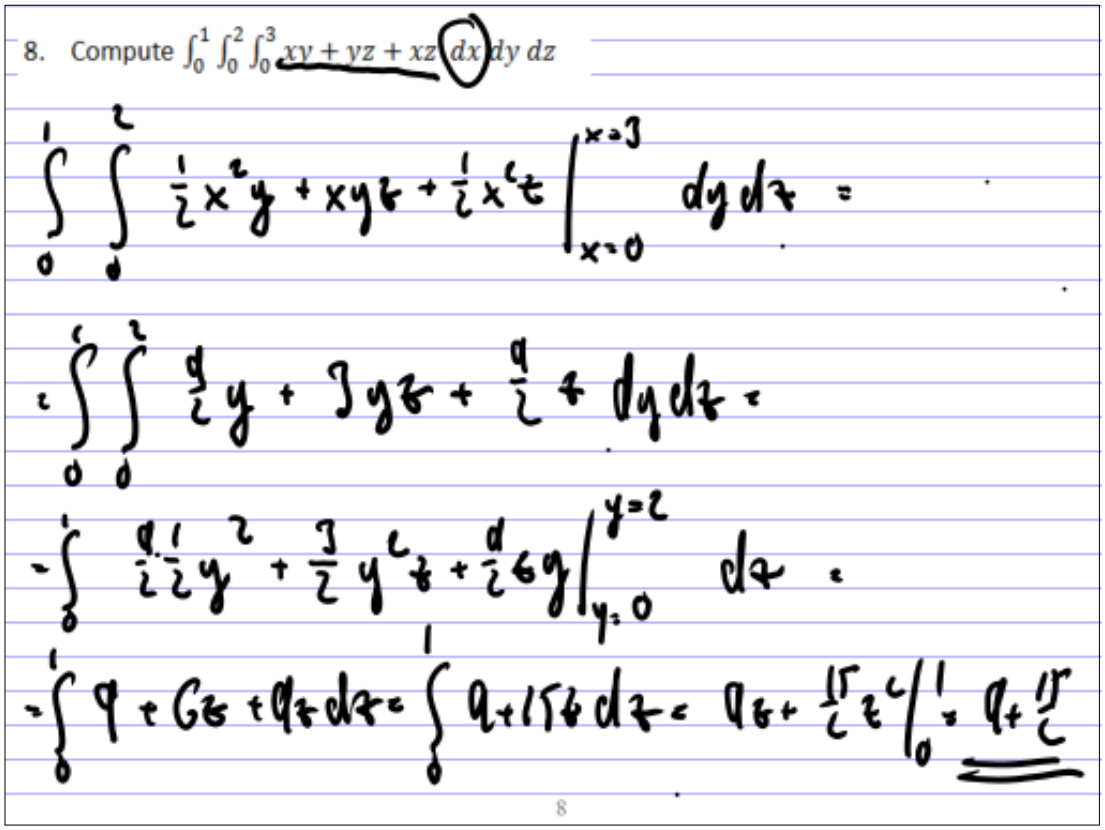
$\frac{1}{3}x^2y^3 + \frac{1}{2}x^3y^2 + Cy + D$

6

Panel 7



Panel 8



Panel 9

9. Find $\int_0^2 \int_1^3 6xy^2\,dy\,dx$ and $\int_1^3 \int_0^2 6xy^2\,dx\,dy$

$$\int_0^2 \int_1^3 6xy^2\,dy\,dx = \int_0^2 6x\tfrac{1}{3}y^3\Big|_{y=1}^{y=3}\,dx =$$

$$= \int_0^2 54x - 2x\,dx = \int_0^2 52x\,dx = \frac{52x^2}{2}\Big|_0^2 = 104$$

9

Panel 10

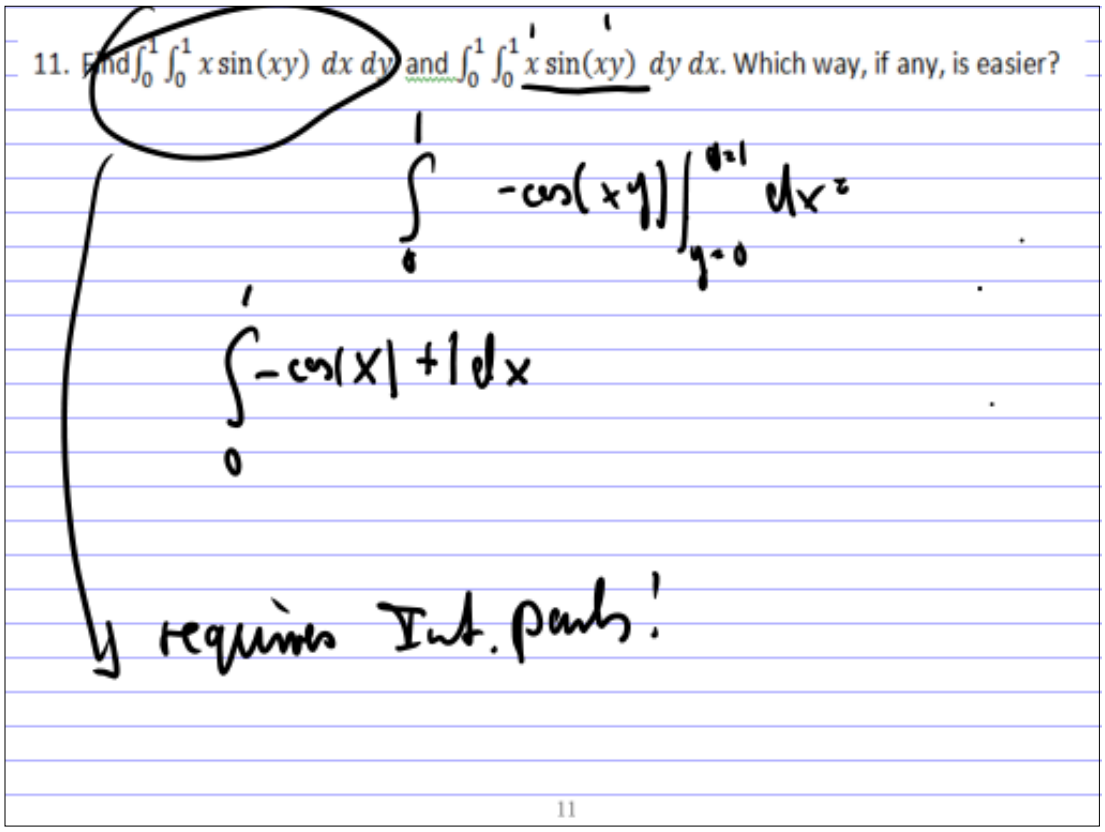
10. Evaluate $\int_e^{e^2} \int_0^1 \frac{x}{y}\,dx\,dy$

$$\int_e^{e^2} \tfrac{1}{2}\frac{x^2}{y}\Big|_{x=0}^{x=1}\,dy = \int_e^{e^2} \tfrac{1}{2}\tfrac{1}{y}\,dy = \tfrac{1}{2}\int_e^{e^2} \tfrac{1}{y}\,dy =$$

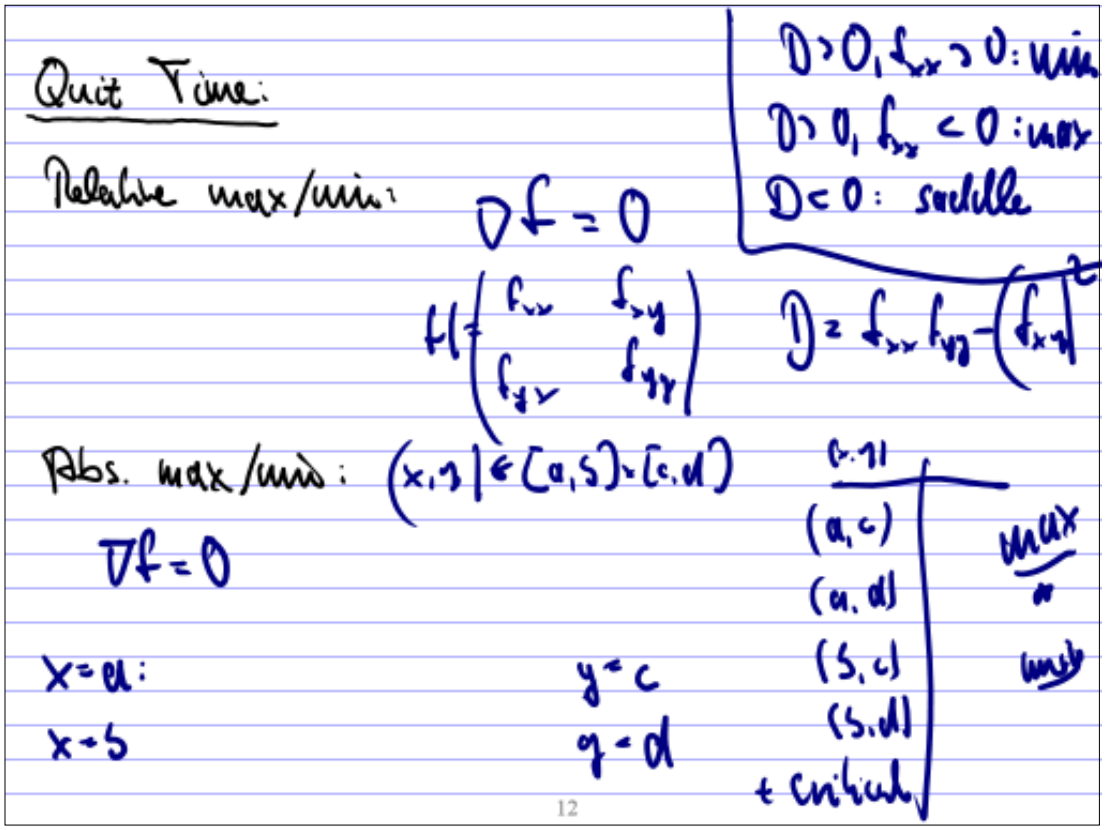
$$= \tfrac{1}{2}\ln|y|\Big|_e^{e^2} = 1 - \tfrac{1}{2} = \tfrac{1}{2}$$

10

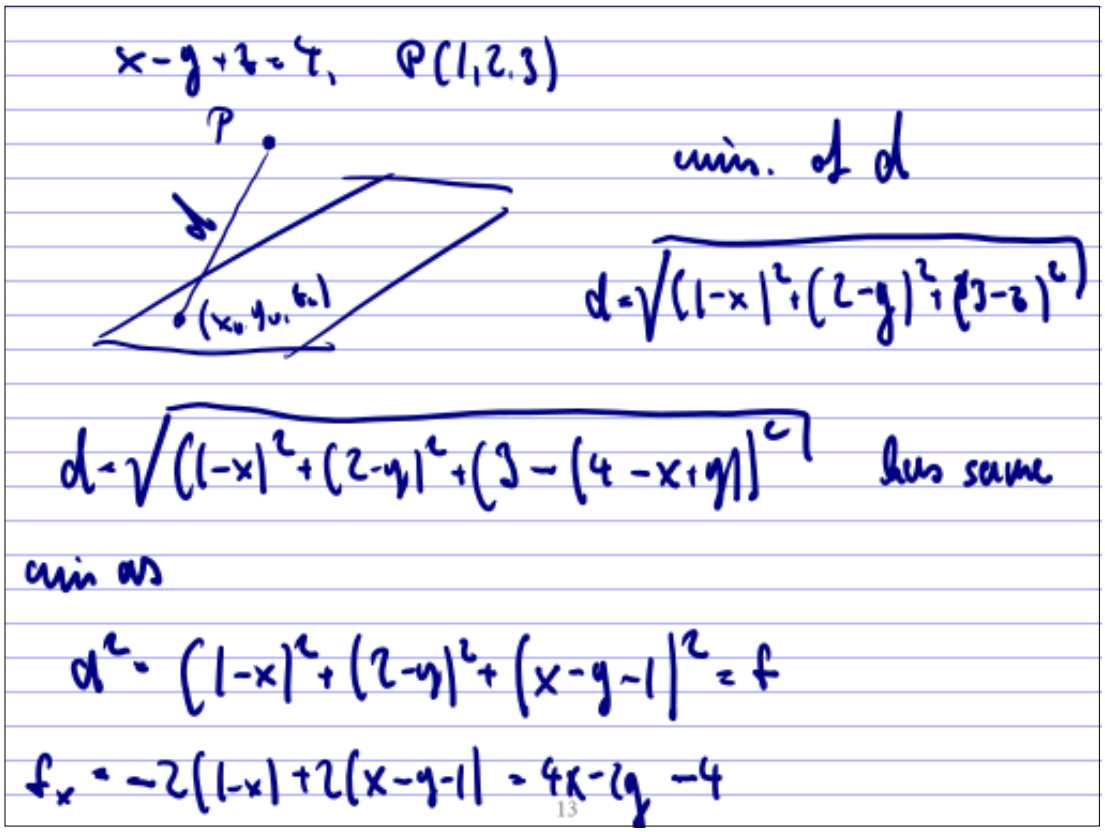
Panel 11



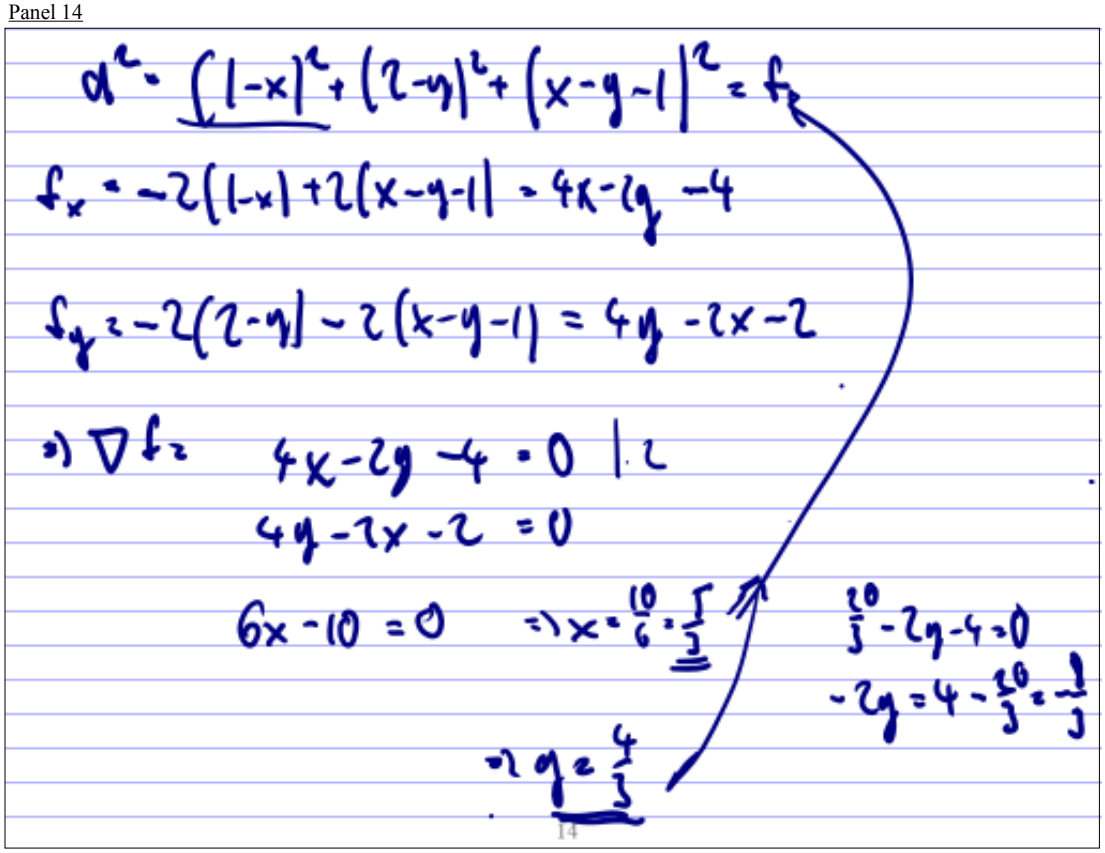
Panel 12



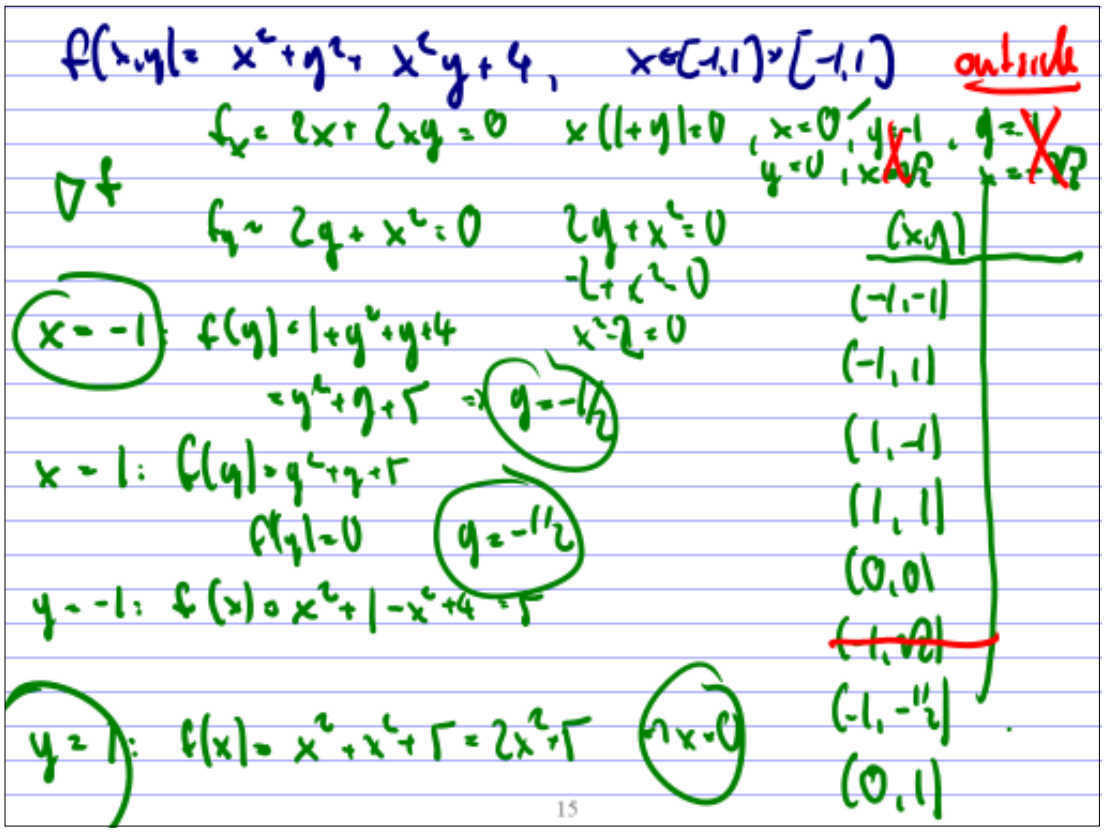
Panel 13



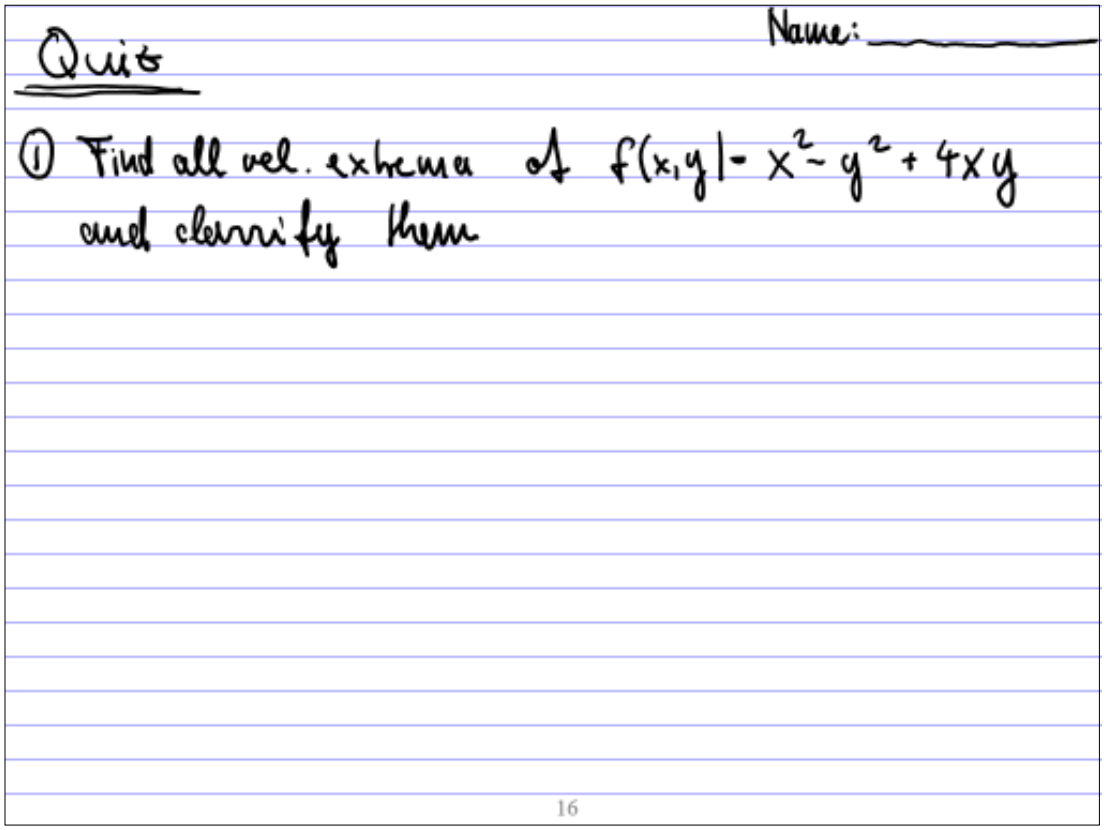
Panel 14



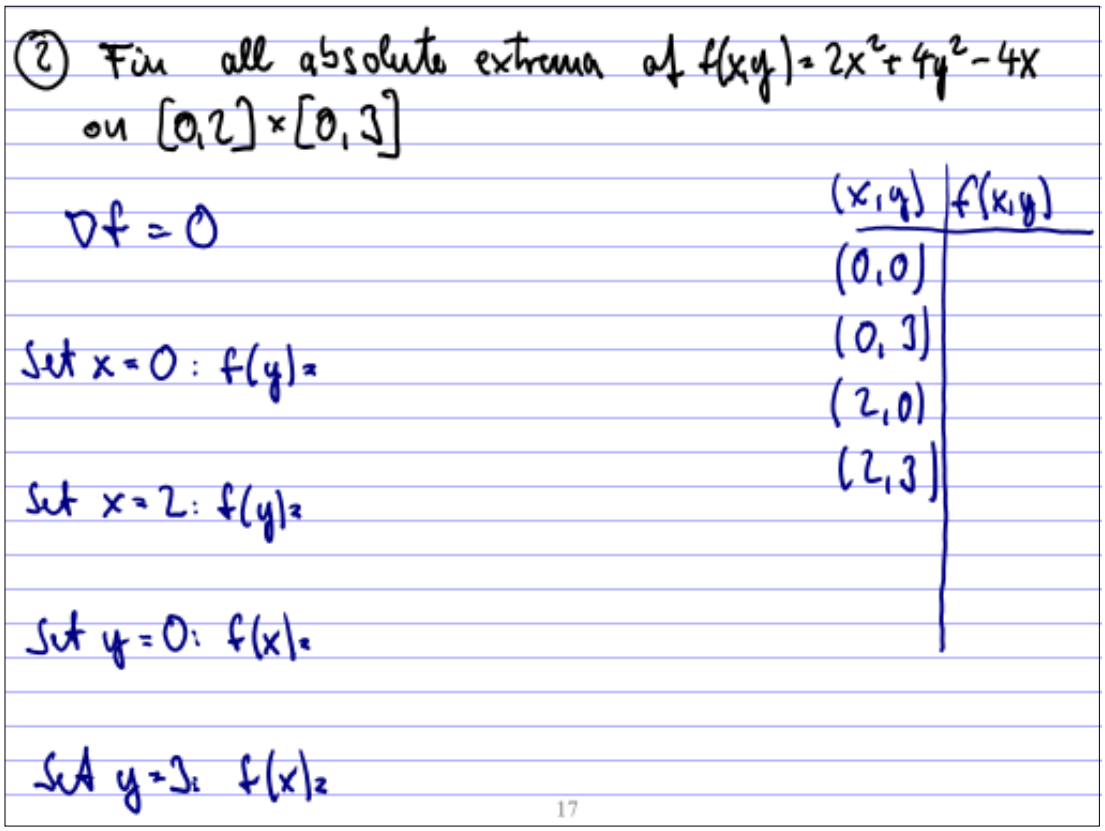
Panel 15



Panel 16



Panel 17


(2) Find all absolute extrema of $f(x,y) = 2x^2 + 4y^2 - 4x$ on $[0,2] \times [0,3]$
$\nabla f = 0$
Set $x = 0$: $f(y) =$
Set $x = 2$: $f(y) =$
Set $y = 0$: $f(x) =$
Set $y = 3$: $f(x) =$
(x,y) | $f(x,y)$
$(0,0)$
$(0,3)$
$(2,0)$
$(2,3)$
17