Panel 1

Quiz 4

Suppose $\vec{r}(t) = \langle t^2, 2, t \rangle$ is a vector-valued function (aka space curve), representing the position of a particle. Find the following:

1. The velocity at $P(0,0,0)$ $r' = v$
2. The speed at $P(0,0,0)$ $\|r'\| = s$
3. The acceleration at $P(0,0,0)$ r''
4. The unit tangent $\vec{T}(t)$ at $P(0,0,0)$ $r'/\|r'\| = v/s$
5. The unit normal vector $\vec{N}(t)$ at $P(0,0,0)$ $T'/\|T'\|$
6. The bi-normal vector $\vec{B}(t)$ at $P(0,0,0)$ $T \times N$
7. The curvature k at $P(0,0,0)$
8. The tangential component of the acceleration a_T at $P(0,0,0)$ $\frac{a \cdot v}{s}$
9. The normal component of the acceleration a_N at $P(0,0,0)$ $\frac{\|a \times v\|}{s}$
10. The osculating plane at $P(0,0,0)$
11. The osculating circle at $P(0,0,0)$

1

Panel 2

$$\int \langle 1,2,0 \rangle dt = \langle \int 1\,dt, \int 2\,dt, \int 0\,dt \rangle =$$

$$= \langle t, 2t, 1 \rangle + D = \langle t+d_1, 2t+d_2, d_3 \rangle$$

$$\vec{r} = \langle t^2, 5t, t^2 - 16t \rangle$$

$$\vec{v} = \langle 2t, 5, 2t-16 \rangle \qquad s = \sqrt{4t^2 + 25 + (2t-16)^2} =$$

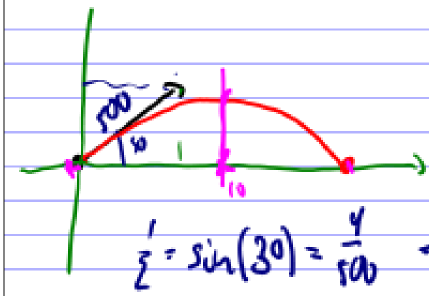
$$= \sqrt{8t^2 - 4t + 25 + 16^2}$$

$$s' = \frac{1}{2}(\quad)^{-1/2}(16t - 4) = 0$$

$$t = 4$$

2

Panel 3



$\vec{a} = \langle 0, -g \rangle$

$\vec{v} = \langle c, -gt + d \rangle$

$\frac{1}{2} = \sin(30) = \frac{y}{500} \Rightarrow y = 250$

$\vec{v}(0) = \langle c, d \rangle = \langle \frac{\sqrt{3}}{2} 500, 250 \rangle$

$x = \frac{\sqrt{3}}{2} \cdot 500$

$\vec{v} = \langle \frac{\sqrt{3}}{2} 500, -gt + 250 \rangle$

$\vec{r}(t) = \langle \frac{\sqrt{3}}{2} 500 t + e, -\frac{1}{2} g t^2 + 250 t + f \rangle$, $\vec{r}(0) = (0, 0)$

$r(t) = \langle \frac{\sqrt{3}}{2} 500 t, -\frac{1}{2} g t^2 + 250 t \rangle$

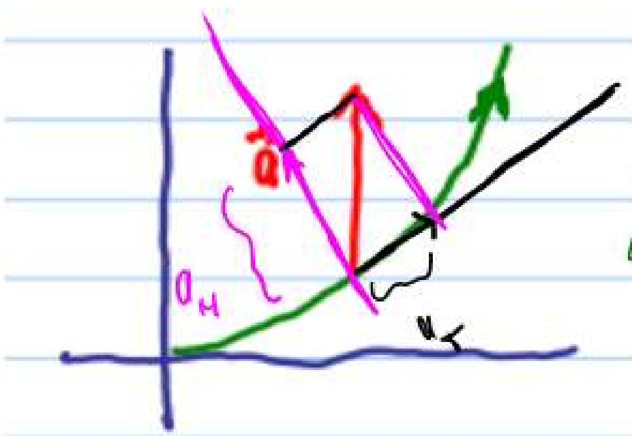
max at $-gt + 250 = 0$

max height: $-\frac{1}{2} g (25)^2 + 250 \cdot 25$, Range: $\frac{\sqrt{3}}{2} 500 \cdot 50$ $\quad t = \frac{250}{g} = 25$

Range: find t s.t. $-\frac{1}{2} g t^2 + 250 t = 0$ $\quad t(250 - \frac{1}{2} g t) = 0$, $t = 0$, $t = 50$

3

Panel 4

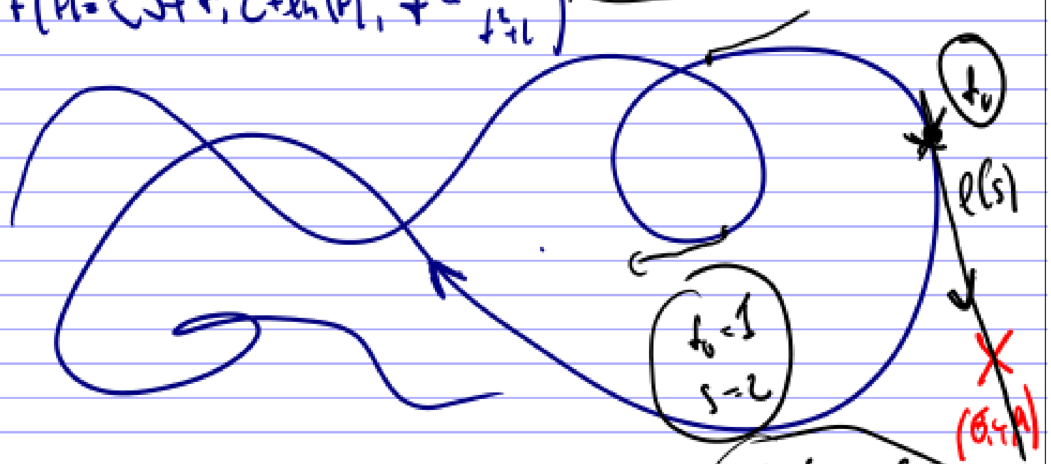


a_T in the dir of movement, speed increases

4

Panel 5

$\vec{r}(t) = \langle 3+t, 2+\ln(t), 7 - \frac{4}{t^2+1} \rangle$ / See find out



$\ell(s) = (6,4,9)$

$r'(t_0)$

$3 + t_0 + s = 6$

$2 + \ln(t_0) + s/t_0 = 4$

$\ell(s) = \langle 3+t_0, 2+\ln(t_0), 7 - \frac{4}{t_0^2+1} \rangle + \langle 1, \frac{1}{t_0}, \frac{4 \cdot 2t}{(t^2+1)^2}$

$S(s) = (6,4,9)$

5

Panel 6

Name: ____________

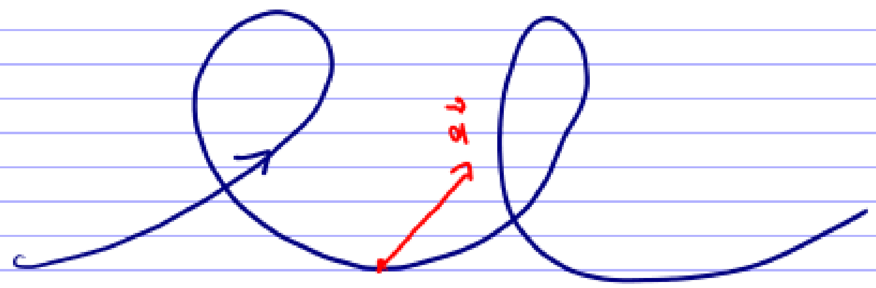
Quiz

(1) If $r(t) = \langle \cos(2t), \sin(2t), 2t \rangle$, find velocity, speed, acceleration, a_T (tangential) and a_N (normal) component of acceleration at $t = \pi$

6

Panel 7

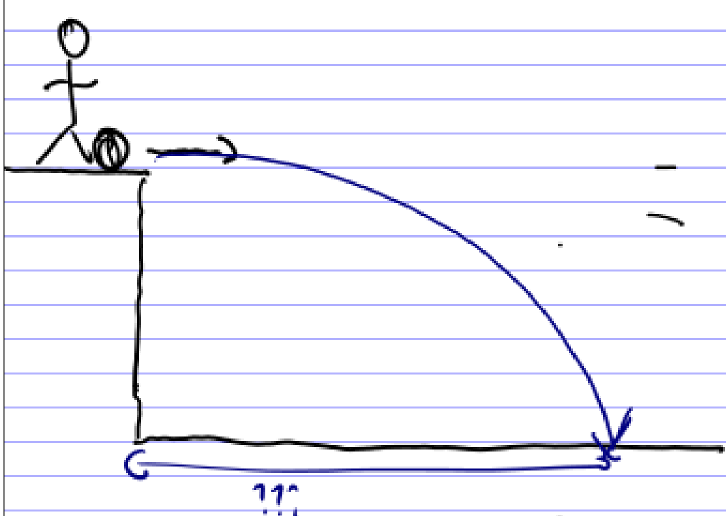
(2) Suppose the acceleration of a particle on a curve $\vec{r}(t)$ is as shown. Sketch a_N and a_T. Also, does the particle speed up or slow down at that time?



7

Panel 8

(3) A girl kicks a soccer ball from a cliff 10m up at a speed of 15 m/sec going horizontally. At what distance does the ball hit the valley floor?



8

Panel 9

Functions of Several Variables

Know: $r: \mathbb{R} \to \mathbb{R}$ e.g. $f(x) = x^2$ Calc I ✓

$r: \mathbb{R} \to \mathbb{R}^2$ e.g. $\vec{r}(t) = \langle t^2, t^3 \rangle$ ✓

$r: \mathbb{R} \to \mathbb{R}^3$ e.g. $\vec{r}(t) = \langle t^2, t^3, t^4 \rangle$

Next: $f: \mathbb{R}^2 \to \mathbb{R}$, e.g. $f(x,y) = xy + x^2 + y^3$

$f: \mathbb{R}^3 \to \mathbb{R}$, e.g. $f(x,y,z) = xy + z^2$

Last: $h: \mathbb{R}^2 \to \mathbb{R}^2$, e.g. $h(x,y) = \cancel{\langle x+y, xy \rangle}$

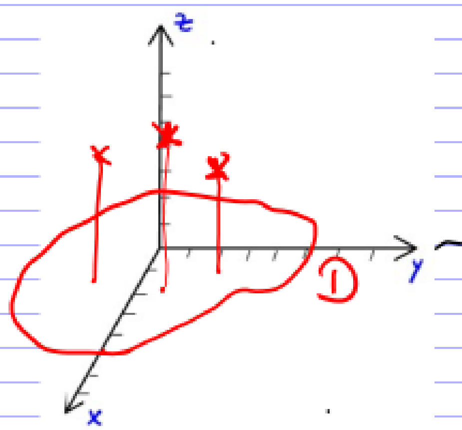
9

Panel 10

Def. A function of 2 variables is a rule that assigns to every pair (x,y) in the domain a unique number z denoted by $f(x,y) = z$

How to visualize $f(x,y) = \boxed{x^2 + y^2 = z}$

x	y	$z = f(x,y)$
-1	-1	2
0	-1	1
1	-1	2
-1	0	1
0	0	0
1	0	1
-1	1	2
0	1	1
1	1	2



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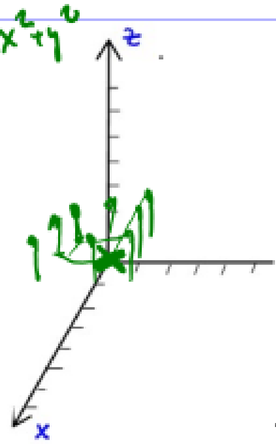
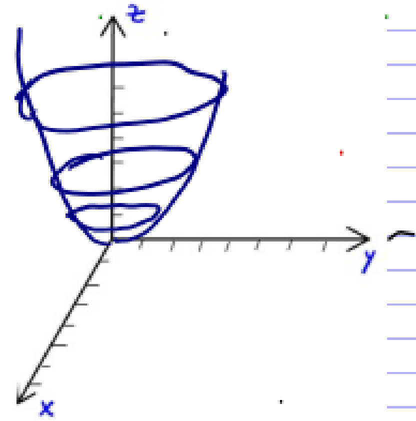
Panel 11

$z = f(x,y) = x^2+y^2$

x	y	$z=f(x,y)$
-1	-1	2
0	-1	1
1	-1	2
-1	0	1
0	0	0
1	0	1
-1	1	2
0	1	1
1	1	2





very hard to visualize!

Need different idea! Fix $z = 1$, what about (x,y)?

$1 = x^2+y^2$ ✓ if $x=0$: $z=y^2$ Trick: look for const. heights

$2 = x^2+y^2$ ✓

Look at $x=0$, $y=0$

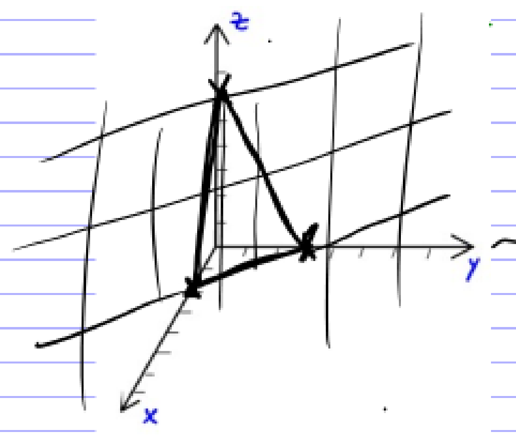
11

Panel 12

Ex: $f(x,y) = 6-3x-2y = z$

$x=0$: $6-2y=z$

$y=0$: $6-3x=z$



$f(x,y) = 6-3x-2y$

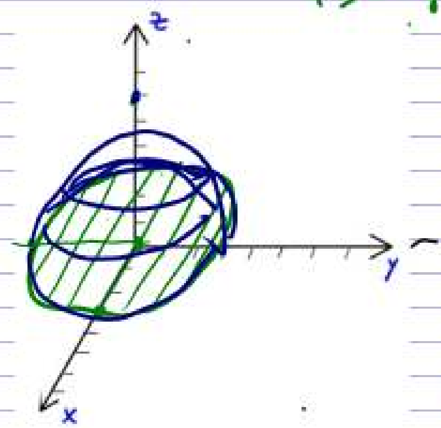
12

Panel 13

Ex: $f(x,y) = \sqrt{9-x^2-y^2} = z$

Worry about domain: $9 - x^2 - y^2 \geq 0$

$9 \geq x^2 + y^2 \leftarrow 9 = x^2 + y^2$



$z = \pm\sqrt{9-x^2-y^2}$

$z^2 = 9 - x^2 - y^2$

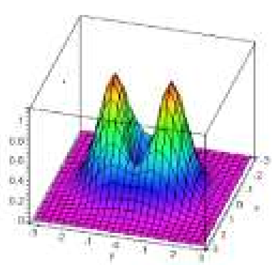
$x^2 + y^2 + z^2 = 9$

upper half of the sphere!

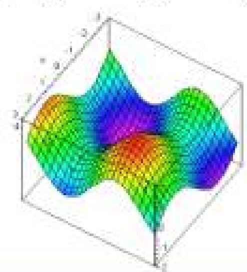
13

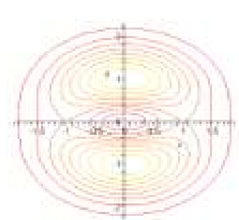
Panel 14

$f(x,y) = (x^2+3y^2)\, e^{-x^2-y^2}$

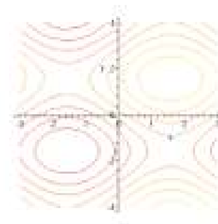


$f(x,y) = \sin(x) + \sin(y)$





Level curves of constant height



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Panel 15

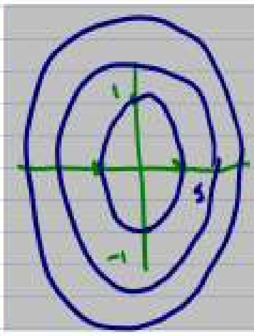
Of course I used Maple to generate these plots

```
> plot3d((x^2+3*y^2)*exp(-x^2-y^2),x=-3..3,y=-4..4);
> plot3d(sin(x)+sin(y),x=-3..3,y=-4..4);
> with(plots);
> contourplot((x^2+3*y^2)*exp(-x^2-y^2),x=-3..3,y=-4..4);
> contourplot(sin(x)+sin(y),x=-3..3,y=-4..4);
>
```

15

Panel 16

Ex: Level curves of $h(x,y) = 4x^2+y^2 = z$



level curves

$4x^2+y^2=1$

$4x^2+y^2=2$

also: $x=0$: $z=y^2$

$y=0$ $z=4x^2$

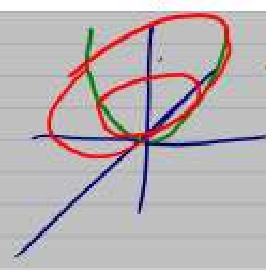
Ex: Level curves for $f(x,y) = 6-3x-2y$

$6-3x-2y=k$

$6-k=3x+2y$

$6-k-3x=2y$

$\frac{6-k}{2}-\frac{3}{2}x=y$

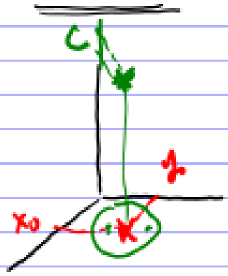


16

h:07

Panel 17

Limits: The limit of $f(x,y)$ as (x,y) approaches (x_0,y_0) is L is written as



$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

Def: Given any $\varepsilon > 0$, there is a $\delta > 0$ s.t.

if $\|(x,y)-(x_0,y_0)\| = \sqrt{(x-x_0)^2+(y-y_0)^2} < \delta$

then $|f(x,y) - f(x_0,y_0)| < \varepsilon$

If I am inside a δ-circle of (x_0,y_0), then $f(x_0,y_0)$ is close to L

17

Panel 18

Ex: $\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2} = \frac{1\cdot 1}{1+1} = \frac{1}{2}$ $\quad \lim_{x\to 0} \frac{x+1}{\cos(x)} = \frac{1}{1} = 1$

l'Hospital does NOT work in $\mathbb{R}^2$

$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \frac{0}{0}$ What to do? $\quad \lim_{x\to 0} \frac{\sin(x)}{x} = 1$

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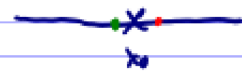
Panel 19

In $\mathbb{R}$:

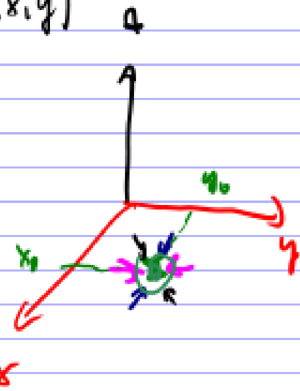
$$\lim_{x \to x_0} f(x)$$



left + right hand limits

In $\mathbb{R}^2$:

$$\lim_{(x,y) \to (x_0,y_0)} f(x,y)$$



inf. many ways to get close to (x_0,y_0),

each way should give same answer

19

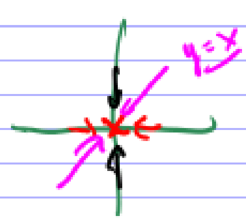
Panel 20

Ex:
$$\lim_{(x,y) \to (1,1)} \frac{xy}{x^2+y^2} = 1/2 \checkmark$$

$$\lim_{(x,y) \to (0,0)} \frac{xy}{x^2+y^2}$$ does not exist

$y=0,\ x \to 0$: $\displaystyle\lim_{(x,0) \to (0,0)} \frac{x \cdot 0}{x^2+0} = 0$

$x=0,\ y \to 0$: $\displaystyle\lim_{(0,y) \to (0,0)} \frac{0 \cdot y}{0+y^2} = 0$



$y = x \to 0$: $\displaystyle\lim_{(x,x) \to (0,0)} \frac{xx}{x^2+x^2} = \lim_{(x,x) \to (0,0)} \frac{\cancel{x^2}}{2x^2} = 1/2$

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