Panel 1

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$\Rightarrow r'(t)$ ✓

$T(t) = \dfrac{r'(t)}{\|r'(t)\|}$

$N(t) = \dfrac{T'(t)}{\|T'(t)\|}$

$B = T \times N$

$\kappa = \|T'(t)\| = \dfrac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$

1

Panel 2

$r(t) = \langle t, \frac{1}{2}t^2, t^2 \rangle$ $\qquad \kappa$ ✓

$r'(t) = \langle 1, t, 2t \rangle$

$r''(t) = \langle 0, 1, 2 \rangle$

$T(t) = \dfrac{1}{\sqrt{1+5t^2}} \langle 1, t, 2t \rangle = (1+5t^2)^{-1/2} \langle 1, t, 2t \rangle$

$T'(t) = -\frac{1}{2}(1+5t^2)^{-3/2}\, 10t \langle 1, t, 2t \rangle + (1+5t^2)^{-1/2} \langle 0, 1, 2 \rangle$

$= (1+5t^2)^{-1/2} \left[\dfrac{-5t}{1+5t^2} \langle 1, t, 2t \rangle + \langle 0, 1, 2 \rangle \right]$

$= (1+5t^2)^{-1/2} \left[\left\langle -\dfrac{5t}{1+5t^2}, \dfrac{\cancel{-5t^2}+1+\cancel{5t^2}}{1+5t^2}, \dfrac{\cancel{-10t^2}+2+\cancel{10t^2}}{1+5t^2} \right\rangle \right]$.

2

Panel 3

$$= (1+5t^2)^{-1/2}\left[\left\langle -\frac{5t}{1+5t^2}, \frac{\cancel{-5t^2}+1+\cancel{5t^2}}{1+5t^2}, \frac{\cancel{-10t^2}+2+\cancel{10t^2}}{1+5t^2}\right\rangle\right].$$

$$= (1+5t^2)^{-3/2}\langle -5t, 1, 2\rangle$$

$$\|r'\| = (1+5t^2)^{-3/2}\|\langle -5t, 1, 2\rangle\| = (1+5t^2)^{-3/2}\sqrt{5+25t^2} =$$

$$= (1+5t^2)^{-1}\cdot\sqrt{5} = \underline{\underline{\frac{\sqrt{5}}{(1+5t^2)}}} = \kappa$$

$$N = \frac{r'}{\|r'\|} = (1+5t^2)^{3/2}(-5t, 1, 2)\cdot\frac{(1+5t^2)}{\sqrt{5}} =$$

$$= \frac{\sqrt{5}}{\sqrt{1+5t^2}}\langle -5t, 1, 2)$$

Panel 4

$$\kappa = \frac{\|r'\times r''\|}{\|r'\|^3} \qquad y = f(x) \qquad \text{let } x = t \Rightarrow y = f(t)$$

$$\Rightarrow r(t) = \langle t, f(t), 0)$$

Works for every function

<u>Other way?</u> $\langle t^3, t^6 + 3t^3 - 1)$

$$\Rightarrow x = t^3, \; y = (t^3)^2 + 3(t^3) - 1$$

$$= x^2 + 3x - 1$$

Works SOMETIMES only!

Panel 5

$r(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$ at $(1,0,0)$ $\Rightarrow t=0$

$r' = \langle -\sin, \cos, \frac{1}{\cos}\cdot(-\sin) \rangle =$

$= \langle -\sin, \cos, -\tan \rangle$

$\|r'\| = \sqrt{1+\tan^2(t)} = \sqrt{\sec^2(t)} = \sec(t) = 1/\cos(t)$

$T = \frac{r'}{\|r'\|} = \langle -\sin(t), \cos(t), -\frac{\sin(t)}{\cos(t)} \rangle \cdot \cos(t) =$

$= \langle -\sin(t)\cos(t), \cos^2(t), -\sin(t) \rangle \Rightarrow T(0) = \langle 0,1,0 \rangle$

$T' = \langle -\cos^2(t) + \sin^2(t), -2\sin(t)\cos(t), -\cos(t) \rangle$ $N(0) = \langle ?, ?, ? \rangle$

$\|T'\| = \checkmark$ $B = T \times N$

Panel 6

Exam

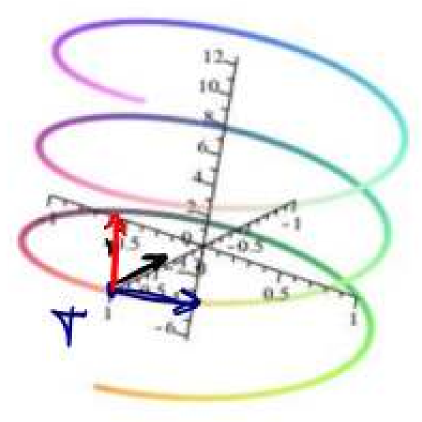
(1) Definition(s)

(a) $\text{proj}_{\vec{w}} \vec{v} = \frac{\vec{v}\cdot\vec{w}}{\|\vec{w}\|^2}$

(2) T/F

(3) Picture problems

(4) Vectors: draw $\vec{v}, \vec{w}$, $\vec{v}+\vec{w}$, $\vec{v}-\vec{w}$, $\vec{v}+2\vec{w}$, or so



Panel 7

$\langle 1,3,-2\rangle, \quad \langle 2,1,2\rangle \qquad P(1,2,3)$

$$\begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 2 & 1 & 2 \end{vmatrix} = \langle 6+2, -(2+4), (1-6)\rangle = \underline{\langle 8,-6,-5\rangle}$$

$$ax+by+cz+d=0$$
$$8x-6y-5z+d=0$$
$$8-12-15+d=0$$
$$-19+d=0$$
$$d=+19$$

$$\Rightarrow \boxed{8x-6y-5z+19=0}$$

7

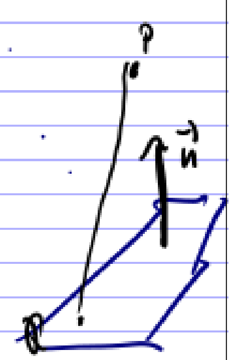
Panel 8

$\ell(t) = P + t\vec{v} \qquad P(1,2,3), \quad Q(1,-1,1)$

$\vec{PQ} = \langle 0,-3,-2\rangle$

$$= (1,2,3) + t(0,-3,-2) =$$
$$= \langle 1, 2-3t, 3-2t\rangle$$

$$d = \begin{cases} \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} & \text{2D line + point} \\ \dfrac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}} & \text{3D plane + point} \\ \|\text{proj}_{\vec{n}}(PQ)\| & \end{cases}$$



8

Panel 9

$x - y + z = 0 \qquad 2x - z = 3$

line of intersection is perp. to both planes, i.e.

perp. to $\langle 1,-1,1\rangle$ and $\langle 2,0,-1\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 0 & -1 \end{vmatrix} = \langle 1, 3, 2\rangle$$

$l(t) = P + t\langle 1,3,2\rangle = \langle 0,-3,-3\rangle + t\langle 1,2,3\rangle$

P: line is passing through yz plane $\Rightarrow x = 0$

$-y + z = 0 \quad y = -3$

$-z = 3 \quad , z = -3$

9

Panel 10

$r(t) = \langle 4t, t^2, t^3\rangle$, $\frac{d}{dt}\|r\| = \frac{d}{dt}\left(\sqrt{16t^2 + t^4 + t^6}\right) =$

$= \frac{1}{2}(\qquad)^{-1/2} \cdot (32t + 4t^3 + 6t^5)$

$\left\|\frac{d}{dt} r\right\| = \|\langle 4, 2t, 3t^2\rangle\|$

$= \sqrt{16 + 4t^2 + 9t^4}$

10

Panel 11

$r(t) = \langle e^t\cos(t), e^t\sin(t)\rangle$

$r'(t) = \langle e^t\cos(t) - e^t\sin(t), e^t\sin(t) + e^t\cos(t)\rangle$

$r'(\pi/2) = \langle -e^{\pi/2}, e^{\pi/2}\rangle$

$T = \frac{r'}{\|r'\|} = \frac{1}{\sqrt{2e^{\pi}}}\langle -e^{\pi/2}, e^{\pi/2}\rangle$.

N = pass but = gives a vector of length 1, perp. to T.

$= \frac{1}{\sqrt{2e^{\pi}}}\langle e^{\pi/2}, e^{\pi/2}\rangle$ -

$\langle 1,2,3\rangle$ is perp. to $\langle 0,3,-2\rangle$

or

$\frac{1}{\sqrt{2e^{\pi}}}\langle -e^{\pi/2}, -e^{\pi/2}\rangle$

11

Panel 12

$T = \frac{r'}{\|r'\|}$, $\|r'\| = 0 \Leftrightarrow x(t) = y(t) = 0 \Rightarrow r$ is <u>not</u> smooth

12