Panel 1

Last Time

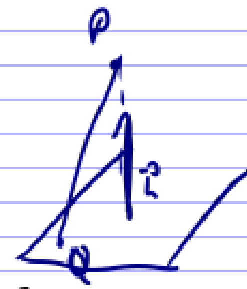
Intersections of (a) lines, (b) planes, (c) plane + line

Distances of

$P(x_0, y_0)$ and line $ax+by+c=0$ in $\mathbb{R}^2$

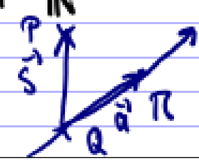
$$d = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$$

$P(x_0, y_0, z_0)$ and plane $ax+by+cz+d$ in $\mathbb{R}^3$



$$d = \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}} = \|\text{proj}_{\vec{n}} \vec{PQ}\|$$

$P(x_0, y_0, z_0)$ and line through Q, R in $\mathbb{R}^3$



$$d = \frac{\|a \times b\|}{\|a\|} \qquad \vec{a} = QR, \ \vec{b} = QP$$

1

Panel 2

Quiz #4

Name: ____________

① True or false:

a) $P(1,2,3)$ is on the plane $3x+2y-z=4$

$x \quad y \quad z$

b) $P(2,3,1)$ is on the line $\ell(t) = \langle 1,1,2\rangle + t\langle 1,2,1\rangle$

② Find the intersection between the line $\ell(t) = \langle -1,0,3\rangle + t\langle 2,-1,1\rangle$ and the plane $x+2y-z=0$

$x = -1+2t$

$y =$

$z =$

solve for t

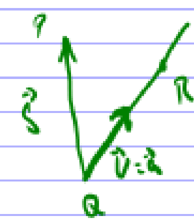
2

Panel 3

(3) Find distance between:

a) $P(2,-5,5)$ and plane $x-2y-2z=0$

b) $P(0,1,0)$ and line $\ell(t)=t\overset{\vec v}{\langle 1,2,-1\rangle}$



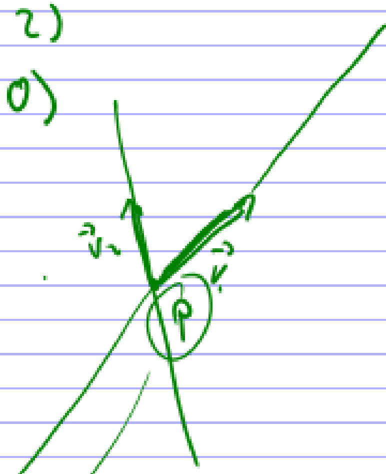
3

Panel 4

$\ell_1(t)=\langle 1,1,0\rangle + t\langle 1,-1,2\rangle$

$\ell_2(s)=\langle 2,0,2\rangle + s\langle -1,1,0\rangle$

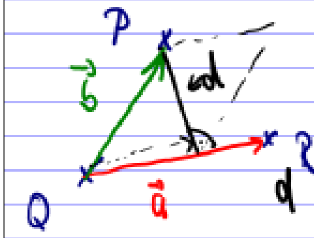
$$\vec n = \vec v_1 \times \vec v_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -1 & 1 & 0 \end{vmatrix}$$



$a x + b y + c z + d = 0$

4

Panel 5



Hint: $a\times(b\times c)=(a\cdot c)b-(a\cdot b)c$

$$d=\left\|b-\text{proj}_{\vec a}(\vec b)\right\| =$$

$$=\left\|b-\frac{a\cdot b}{\|\vec a\|^2}a\right\| = \left\|\frac{\|a\|^2 b-(a\cdot b)a}{\|a\|^2}\right\| =$$

$$=\left\|\frac{(a\cdot a)b-(a\cdot b)a}{\|a\|^2}\right\| = \frac{\|a\times(b\times a)\|}{\|a\|^2} =$$

$$=\frac{\cancel{\|a\|}\cdot\|a\times b\|\cdot\sin(\theta)}{\|a\|^2} = \frac{\|a\times b\|}{\|a\|}$$

$\pi/2$

Prove the hint. $\vec a\times(\vec b\times\vec c)=(a\cdot c)b-(a\cdot b)c$ by writing $a=\langle a_1,a_2,a_3\rangle$, $b=\langle b_1,b_2,b_3\rangle$, $c=\langle c_1,c_2,c_3\rangle$

Panel 6

Name: ____________

Quiz #4

1. True or false:

   a) $P(1,2,3)$ is on the plane $3x+2x-z=4$

   b) $P(2,3,1)$ is on the line $\ell(t)=\langle 1,1,2\rangle+t\langle 1,2,1\rangle$

2. Find the intersection between the line $\ell(t)=\langle -1,0,3\rangle+t\langle 2,-1,1\rangle$ and the plane $x+2y-z=0$

Panel 7

Chapter 10 Review

Started with $\mathbb{R}^3$ points, spheres, sheets

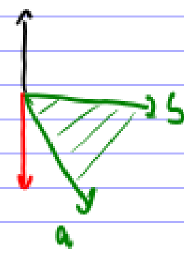
Vectors: add, subtract, mult. by scalar, length

Dot product: angles, perp, projection

Cross product: perp. to both vectors, length = area of parallel.

"right-hand rule" applies

$\vec{a}\times\vec{b}$



rotate right hand from a to b

$\Rightarrow$ thumb shows dir. of cross pr.

Lines: ✓

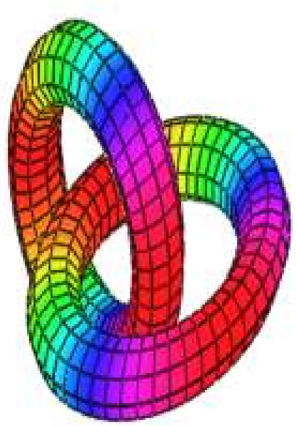
Planes: ✓

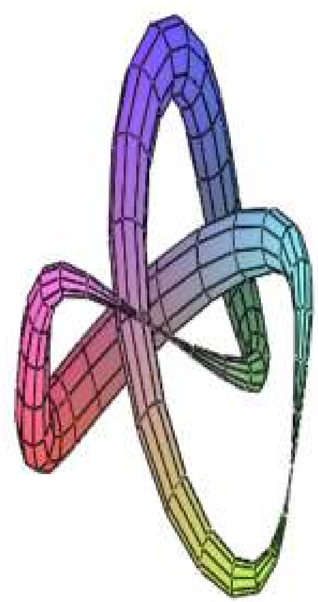
Distances + Intersections

7

Panel 8

Space Curves





8

Panel 9

Space Curves

or $r(t) = \langle f(t), g(t) \rangle$

Def: $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ is a vector-valued "function" with component functions f, g, and h

Many concepts work as they should: If $\vec{r}(t)$ is vector-valued function then

Limit: $\lim_{t \to t_0} \vec{r}(t) = \left\langle \lim_{t \to t_0} f(t), \lim_{t \to t_0} g(t), \lim_{t \to t_0} h(t) \right\rangle$

Derivative: $\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$

Integral: $\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b f(t)\,dt, \int_a^b g(t)\,dt, \int_a^b h(t)\,dt \right\rangle$

9

Panel 10

Ex: $r(t) = \left\langle \frac{t^2 - 2t}{t}, \frac{\cos(t)}{t}, \frac{\sin(t)}{t} \right\rangle$

Find $\lim_{t \to 0} \vec{r}(t) = \left\langle \lim_{t \to 0} \frac{t^2 - 2t}{t}, \lim_{t \to 0} \frac{\cos(t)}{t}, \lim_{t \to 0} \frac{\sin(t)}{t} \right\rangle$

is undefined

$\lim_{t \to 0} \frac{2t - 2}{1}$ $\qquad$ $\lim_{t \to 0} \frac{\cos(t)}{1}$

$\langle -2, \text{undef}, 1 \rangle$

$r(t) = \left\langle \underbrace{t^3 + \frac{1}{\sqrt{t}} + \pi^2}, t \cdot \ln(t), \frac{e^{x^2}\cos(x)}{\sin(x)} \right\rangle$

$t^{-1/2}$

Find $r'(t) = \langle 3t^2 - \frac{1}{2}t^{-3/2} + 0, \; 1\ln(t) + t \cdot \frac{1}{t} = \ln(t) + 1,$

$\frac{\left(2xe^{x^2} \cdot \cos(x) + e^{x^2}(-\sin(x))\right)\sin(x) + e^{x^2}\cos(x) \cdot \cos(x)}{[\sin(x)]^2}$

10

Panel 11

The problem: with vector-valued functions is to visualize them, and interprete the deriv. + integrals:

Ex: $\vec{r}(t) = \langle 1+t, 2+5t, -1+6t \rangle$ - describe graph

line through $\langle 1,2,-1 \rangle$ in dir $\langle 1,5,6 \rangle$
$t=0$

Ex: $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$ - describe graph
$x \quad y \quad z$

spiral around z-axis

$x = \cos(t)$
$y = \sin(t)$
$z = t$

$x^2 + y^2 = 1$

11

Panel 12

Sketch graph of

$r_1(t) = \langle (4+\sin(20t))\cos(t), (4+\sin(20t))\sin(t), \cos(20t) \rangle$

$r_2(t) = \langle (2+\cos(1.5t))\cos t, (2+\cos(1.5t))\sin(t) \quad \sin(1.5t) \rangle$

Wolfram Alpha:

ParametricPlot3D[, ,]

12

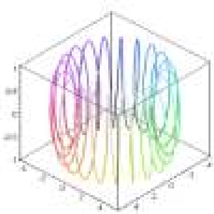
Panel 13

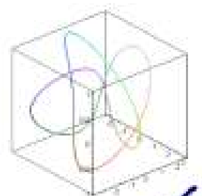
Sketch graph of

$r_1(t) = \langle (4+\sin(20t))\cos(t), (4+\sin(20t))\sin(t), \cos(20t) \rangle$

$r_2(t) = \langle (2+\cos(1.5t))\cos t, (2+\cos(1.5t))\sin(t)\ \ \sin(1.5t) \rangle$

```
> with(plots):
> spacecurve( [(4+sin(20*t))*cos(t), (4+sin(20*t))*sin(t), cos(20*t)], t=0..2*Pi, numpoints=500);
> spacecurve( [(2+cos(1.5*t))*cos(t), (2+cos(1.5*t))*sin(t), sin(1.5*t)], t=0..4*Pi, numpoints=500);
> spacecurve([t, t^2, t^3], t=0..2);
```





could use "tube plot"

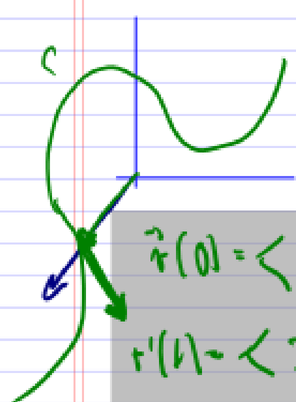
also plot3d / implicitplot3d

Panel 14

Derivatives of Space Curves aka Vector-valued functions

If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ and f, g, h are differentiable

then $\vec{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$



Ex: $\vec{r}(t) = \langle 1+t^3, te^{-t}, \sin(2t) \rangle$

Find $\vec{r}(0)$ and $\vec{r}'(0)$ ← tangent vector

Compute $\vec{r}(0) \cdot \vec{r}'(0)$

$\vec{r}(0) = \langle 1, 0, 0 \rangle$

$r'(t) = \langle 3t^2, e^{-t} - te^{-t}, 2\cos(2t) \rangle$

$r'(0) = \langle 0, 1, 2 \rangle$

Panel 15

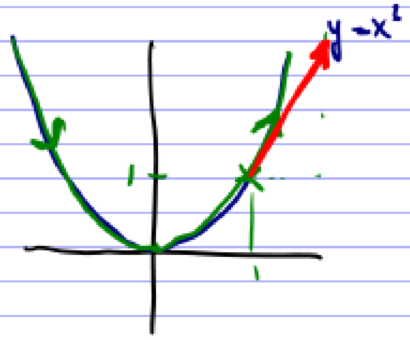
$r(t) = \langle t, t^2 \rangle$ Find $\vec{r}(1)$, $\vec{r}'(1)$ and graph $\vec{r}'(1)$

$\dot{x}$ $\ddot{y}$

$x = t,\ y = t^2 = x^2$

$r'(t) = \langle 1, 2t \rangle$

$r'(1) = \langle 1, 2 \rangle$



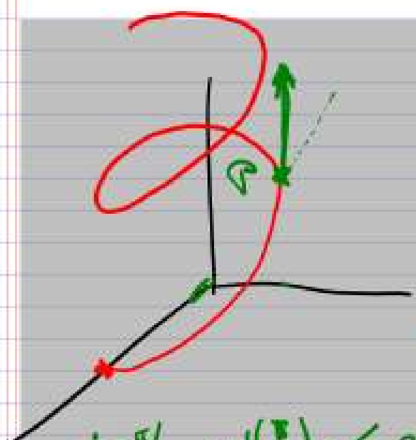
$\vec{r}'(t_0)$ is the vector tangent to curve through point $\vec{r}(t_0)$

15

Panel 16

Ex. Find equation of tangent line to $r(t) = \langle 2\cos t, \sin t, t \rangle$ at the point $P(0, 1, \pi/2)$

$r(0) = \langle 2, 0, 0 \rangle$



$r'(t) = \langle -2\sin(t), \cos(t), 1 \rangle$

what t gives $r(t) = P$, i.e.

$\langle 2\cos(t), \sin(t), t \rangle = (0, 1, \tfrac{\pi}{2})$

$t = \frac{\pi}{2}$: $r'(\frac{\pi}{2}) = \langle -2, 0, 1 \rangle$

16

Panel 17

Integrals of Space Curves aka vector valued functions

If $r(t) = \langle f(t), g(t), h(t) \rangle$ and f, g, h are integrable

then $\int \vec{r}(t)\,dt = \left\langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \right\rangle$

Ex: If $\vec{r}(t) = 2\cos(t)\,\vec{i} + \sin(t)\,\vec{j} + 2t\,\vec{k}$, find $\int_0^{\pi/2} \vec{r}(t)\,dt$

$r(t) = \langle 2\cos(t), \sin(t), 2t \rangle$

$$\int_0^{\pi/2} \vec{r}(t)\,dt = \left\langle \int_0^{\pi/2} 2\cos(t)\,dt, \int_0^{\pi/2} \sin(t)\,dt, \int_0^{\pi/2} 2t\,dt \right\rangle$$

$$= \left\langle 2\sin(t)\Big|_0^{\pi/2}, -\cos(t)\Big|_0^{\pi/2}, t^2\Big|_0^{\pi/2} \right\rangle = \left\langle 2, 1, \left(\tfrac{\pi}{2}\right)^2 \right\rangle$$

17

Panel 18

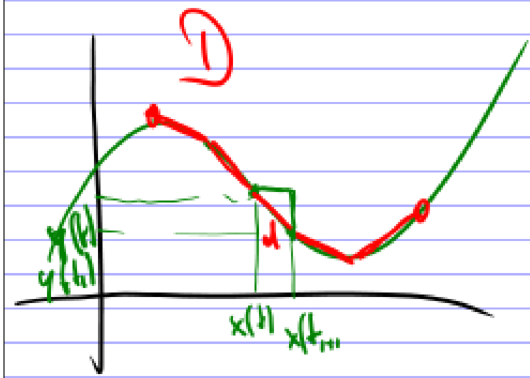
Arc Length (or just "length" of a curve

If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ then $L =$

Ex: Find length of $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$, $t = 0$ to 2π

18

Panel 19



$r(t) = \langle x(t), y(t) \rangle$

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$D = \sum \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

summation

$$D \approx \sum \sqrt{(\Delta t)^2\left(\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2\right)} = \sum \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2}\,\Delta t$$

$$D = \lim_{\Delta t \to 0} \int_a^b \sqrt{(x')^2 + (y')^2}\,dt = ?$$

$$\lim_{t \to t_0} \frac{x(t) - x(t_0)}{t - t_0} = x'(t)$$

19