Panel 1

Last time

Dot Product: $a \cdot b = \|a\| \|b\| \cdot \cos\theta$

Cross Product: $\|a \times b\| = \|a\| \|b\| |\sin\theta|$ ← area of parallelogram

Parametric Equation of line:

$\ell(t) = P_0 + t\vec{v} = \langle x_0, y_0, z_0 \rangle + t\langle v_1, v_2, v_3 \rangle = \langle x_0 + tv_1, y_0 + tv_2, z_0 + tv_3 \rangle$

Scalar Equation of a plane: $\sim P_0 + t\vec{v} + s\vec{w}$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$ax + by + cz = d$

$\vec{n} = \langle a, b, c \rangle$ is normal (perp.) to plane.

1

Panel 2

Take line $\ell(t) = \langle 3, -2, -4 \rangle + t \cdot \langle 1, 2, 3 \rangle$
and plane $3x - 2y + 4z + 2 = 0$

a) Find any 2 points on ℓ $(3, -2, -4)$ and $(4, 0, -1)$ $(t=1)$ ...

is $P(1,2,3)$ on ℓ? If so...

$\ell(t) = (1,2,3) = (\underbrace{3+t}_{t=-2}, -2+2t, -4+3t)$ does not match 2nd comp. so NO.

b) Find any 2 points on plane

$(0, 0, -\frac{1}{2})$, $(0, 1, 0)$, $(-\frac{2}{3}, 0, 0)$, $(1, 2, -\frac{1}{4})$

Is $P(1,2,3)$ on the plane?

$3 \cdot 1 - 2 \cdot 2 + 4 \cdot 3 + 2 \neq 0$ No

2

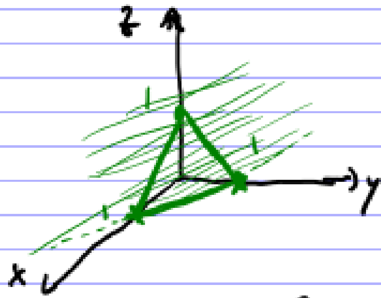
Panel 3

Graphing Planes: Planes can be visualized by looking at the traces in the coordinate planes.

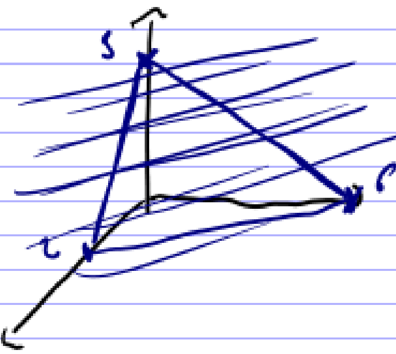
$x + y + z = 1$

$z = 0$: $x + y = 1$, $y = 1 - x$

$y = 0$: $x + z = 1$



$2x + 5y + z = 10$



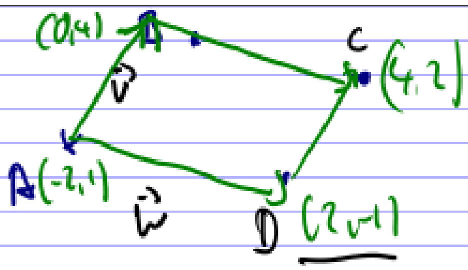
$3x + y + 2z = 6$

$x = 0$: $y + 2z = 6$

$y = 0$: $3x + 2z = 6$

3

Panel 4



$\vec{v} = \vec{AB} = \langle 0-(-2), 4-1 \rangle$
$= \langle 2, 3 \rangle$

$\vec{w} = \vec{AD} = \langle 4, -2 \rangle$

$\vec{v} = \langle 2, 3 \rangle \cong \langle 2, 3, 0 \rangle$ embed $\mathbb{R}^2$ into $\mathbb{R}^3$

$\vec{w} = \langle 4, 2 \rangle \cong \langle 4, -2, 0 \rangle$

$$\begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 4 & -2 & 0 \end{vmatrix} = \langle 0, 0, -4-12 \rangle = \langle 0, 0, -16 \rangle$$

$$\text{area} = \| \langle 0, 0, -16 \rangle \| = \sqrt{16^2} = \underline{\underline{16}}$$

4

Panel 5

$P(1,2,3), Q(-4,3,0)$

$\ell(t) = \langle 1,2,3\rangle + t\cdot\vec{PQ} = \langle 1,2,3\rangle + t\cdot\langle -4-1, 3-2, 0-3\rangle$

$= \langle 1,2,3\rangle + t\langle -5,1,-3\rangle$

$\langle 1+2t, 3t, 2-t\rangle$ and $\langle -1+s, 4+s, 1+3s\rangle$ intersect?

$1+2t = -1+s \Rightarrow 2+2t = s$

$3t = 4+s \quad 3t = 4+2+2t \Rightarrow t=6,\ s=14$

$2-t = 1+3s \quad 2-6 \overset{?}{=} 1+3\cdot 14$ NO skew

5

Panel 6

4c) $\vec{PQ}\times\vec{PR} = \langle -13, 17, 7\rangle = \langle a,b,c\rangle = \vec{n}$

$-13x + 17y + 7z = d$

$-13(x+1) + 17(y+2) + 7(z+3) = 0$

foil out $\Rightarrow$ D

$-13(x-8) + 17(y-2) + 7(z-4) = 0$

foil out $\rightarrow$ D

6

Panel 7

Name: ____________

Quiz # 3

① Find the equation of a line through $P(1,2,0)$ and $Q(3,6,1)$.

point + f. direction

② Find equation of plane through $P(1,2,0)$, $Q(3,6,1)$, and $R(0,0,2)$

$\vec{n} = \vec{PQ} \times \vec{PR} \Rightarrow$

$a(x-\_) + b(y-\_) + c(z-\_) = 0$

7

Panel 8

③ Are the following lines parallel only?

$\langle 0,1,0\rangle + t\langle 1,2,3\rangle$ and $\langle 1,-1,1\rangle + s\langle -1,3,2\rangle$.

④ If two lines in $\mathbb{R}^3$ are not parallel, do they have to intersect?

⑤ Find the angle between the planes

$x + 2y + 3z = 0$ and $2x - 4y + 2z = 5$

$\vec{n}_1 = \langle \quad \rangle$ $\qquad$ $\vec{n}_2 = \langle \quad \rangle$

$\Rightarrow \cos(\theta) = \dfrac{n_1 \cdot n_2}{\|n_1\|\|n_2\|}$

8

Panel 9

9

Back at 12:00pm!

Panel 10

http://www.slideshare.net/leingang/lesson-4-lines-planes-and-the-distance-formula

10

Panel 11

Intersections:

Find intersection of $\ell(t) = \langle 1,2,3\rangle + t\langle 1,1,1\rangle$ and
$2x - y + z = 0$

Find intersection of $\ell(t) = \langle 0,1,0\rangle + t\langle 1,0,1\rangle$ and
$\ell(s) = \langle -1,-2,1\rangle + t\langle 2,3,0\rangle$

Find intersection of $-x + y + z = 0$ and
$2x - y + z = 1$

11

Panel 12

Find intersection of $\ell(t) = \langle 1,2,3\rangle + t\langle 1,1,1\rangle$ and
$2x - y + z = 0$

Line + plane are either parallel or they intersect.

Parallel? $\vec{n} = \langle 2,-1,1\rangle \perp \vec{v} = \langle 1,1,1\rangle$

$\langle 2,-1,1\rangle \cdot \langle 1,1,1\rangle = 2$ Not parallel

point on line $(\underset{x}{1+t}, \underset{y}{2+t}, \underset{z}{3+t})$

$2(1+t) - (2+t) + (3+t) = 0$

$3 + 2t = 0 \Rightarrow t = -\frac{3}{2}$

$\ell(-\frac{3}{2}) = \langle 1-\frac{3}{2}, 2-\frac{3}{2}, 3-\frac{3}{2}\rangle$

$= \langle -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\rangle$

12

Panel 13

Find intersection of $\ell(t) = \langle 0,1,0\rangle + t\langle 1,0,1\rangle$ and
$\ell(t) = \langle -1,-2,1\rangle + t\langle 2,3,0\rangle$

$$\langle 0,1,0\rangle + t\langle 1,0,1\rangle = \langle -1,-2,1\rangle + s\langle 2,3,0\rangle$$

finally!

$$0 + 1t = -1 + 2s$$
$$\boxed{1 + 0 = -2 + 3s} \qquad s = 1$$
$$0 + \boxed{t = 1} \qquad t = 1$$

They intersect in point $\ell(1) = \langle 1,1,1\rangle$

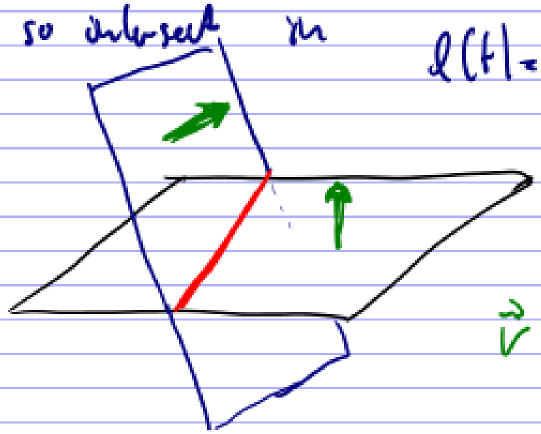
13

Panel 14

$-x + y + z = 0$ and $2x - y + z = 1$ are either parallel or they intersect in a line!

$n_1 = \langle -1,1,1\rangle$ $\qquad n_2 = \langle 2,-1,1\rangle$ not parallel

so intersect in $\ell(t) = P_0 + t\vec{v}$



The direction of intersection is perp. to both planes!

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = \langle 2,3,-1\rangle$$

On that line $z=0$: $-x+y=0$, $2x-y=1$ $\Rightarrow x=1, y=1$ $\Rightarrow \ell(t) = \langle 1,1,0\rangle + t\langle 2,3,-1\rangle$

14