Panel 1

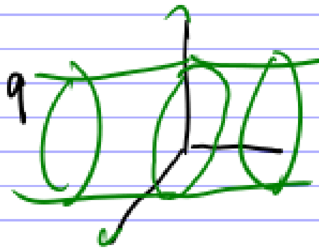
Last time

Coordinate system in $\mathbb{R}^3$

Distance formula

3D Objects

$$x^2+y^2+z^2-4z=9$$
$$x^2+z^2=4$$

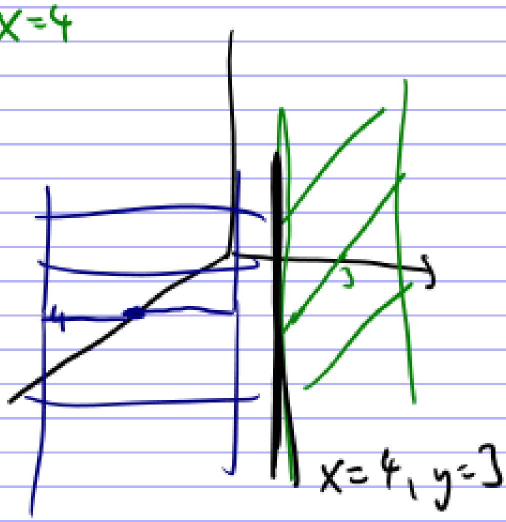


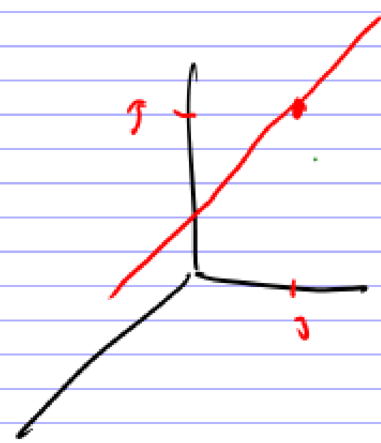
Vectors:
- add, subtract
- mult. by #'s
- length
- unit vectors

1

Panel 2

$x=4$ $y=3$





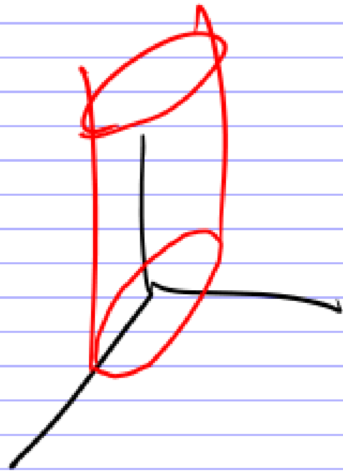
$y=3,\ z=5$: is a line

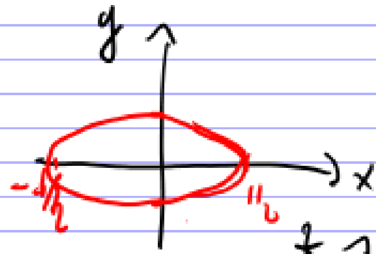
through $y=3$, $5=z$, parallel to x-axis

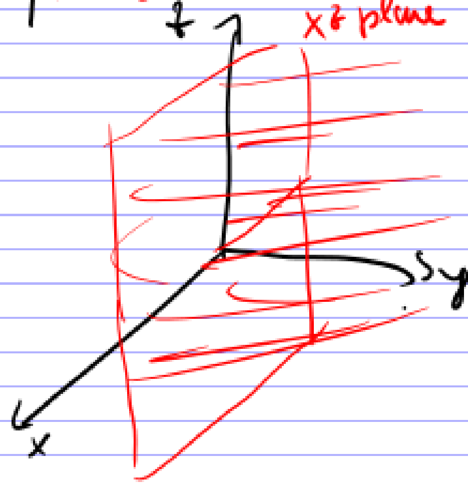
2

Panel 3

$4x^2+9y^2=1$



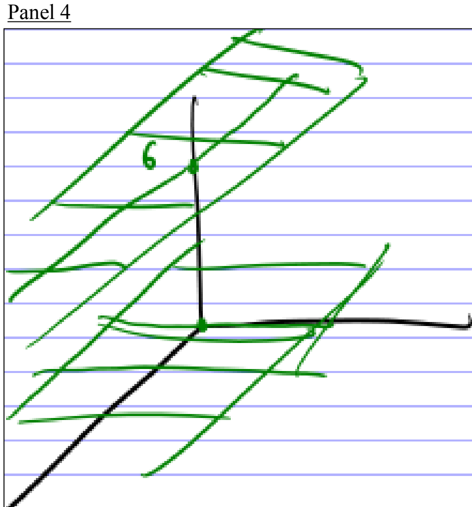




$y=0$, $y \geq 0$

3

Panel 4



$x^2+y^2+z^2 \leq 3$

solid ball , radius $\sqrt{3}$

$\vec{v} = \langle -4, 2, 4 \rangle$, $\|\vec{v}\| = \sqrt{16+4+16} = 6$

$\vec{u} = \frac{1}{6} \langle -4, 2, 4 \rangle$

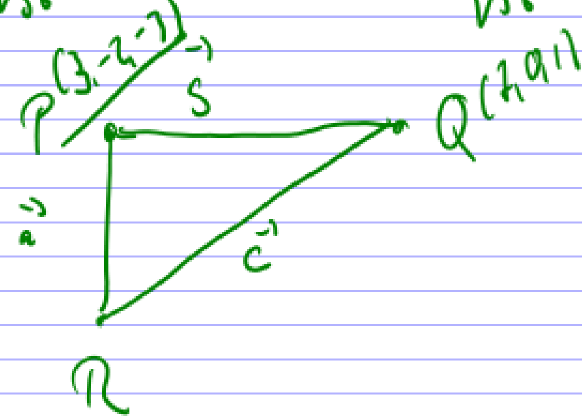
4

Panel 5

$\vec{v} = \langle 2,4,6 \rangle$. Same dir., length 15

$\|\vec{v}\| = \sqrt{4+16+36} = \sqrt{56}$

$\frac{15 \cdot 1}{\sqrt{56}} \cdot \langle 2,4,6 \rangle = \frac{15}{\sqrt{56}} \langle 2,4,6 \rangle$



$\|PQ\| = \|\langle 4,2,4 \rangle\| = 6$

$\|PR\| = 5$

$\|RQ\| = 4$

5

Panel 6

Calc 3 - Quiz #1

① Find the distance between $P(-1,2,0)$ and $Q(2,1,1)$.

② Find radius of sphere $x^2+y^2+z^2-6x+4y-2z=11$

6

Panel 7

③ Describe 3D object given by $x^2+z^2=4$

④ Find a vector in direction $\langle -3,4,5 \rangle$ with length 2.

7

Panel 8

Other ways to describe vectors: Find a vector of length 2 that makes an angle of $\pi/3$ with positive x-axis.

$x = 2\cdot\cos(\pi/3) = 2\cdot 1/2 = 1$

$y = 2\cdot\sin(\pi/3) = 2\cdot \sqrt{3}/2 = \sqrt{3}$

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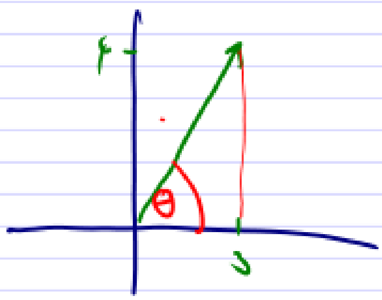
Panel 9

Other way around: find angle that $\vec{v} = 3\vec{i} + 4\vec{j}$ makes with the positive x-axis.

$\vec{i} = \langle 1,0 \rangle$

$\vec{j} = \langle 0,1 \rangle$

$3\vec{i} + 4\vec{j} = 3\langle 1,0 \rangle + 4\langle 0,1 \rangle = \langle 3,0 \rangle + \langle 0,4 \rangle = \langle 3,4 \rangle$



$\cos(\theta) = \frac{x}{5}$

$\sin(\theta) = \frac{y}{5}$

$\frac{\sin\theta}{\cos\theta} = \frac{y/5}{x/5} = \frac{y}{x} = \tan(\theta) =$

$\theta = \arctan\left(\frac{4}{3}\right) = 53^\circ$

$= \tan(\theta) = \frac{4}{3}$

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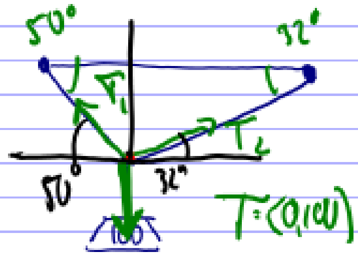
Panel 10

What are vectors good for: a 100 lb weight hangs from two wires as shown. Find the forces T_1 and T_2 acting on the wires and their magnitudes.



$T_1 = \|T_1\| \langle -\cos(50), \sin(50) \rangle$

$T_2 = \|T_2\| \langle \cos(32), \sin(32) \rangle$

$T_1 + T_2 - T = 0 \qquad T_1 + T_2 = \langle 0, 100 \rangle$

$-\|T_1\|\cos(50) + \|T_2\|\cos(32) = 0 \Rightarrow \|T_1\| = \|T_2\| \frac{\cos(32)}{\cos(50)}$

$\|T_1\|\sin(50) + \|T_2\|\sin(32) = 100$

HW

10

Panel 11

Check as HW Act $T_1 \approx \langle -55.05, 65.6 \rangle$

$T_2 = \langle 55.05, 34.4 \rangle$

11

Panel 12

Know how to add / subtract vectors. How to multiply?

Ex: $\langle 3,2 \rangle + \langle 1,5 \rangle = \langle 4,7 \rangle$

why not: $\langle 3,2 \rangle \cdot \langle 1,5 \rangle \overset{?}{=} \langle 3\cdot 1, 2\cdot 5 \rangle = \langle 3,10 \rangle$

No good: $\langle 1,0 \rangle \cdot \langle 0,5 \rangle = \langle 0,0 \rangle$

but neither vector is zero-vector $\Rightarrow$ No good

Dot Product

(2D) $\vec{v} \cdot \vec{w} = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = v_1 w_1 + v_2 \cdot w_2$

(3D) $= \langle v_1, v_2, v_3 \rangle \cdot \langle w_1, w_2, w_3 \rangle = v_1 w_1 + v_2 w_2 + v_3 w_3$

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Panel 13

Examples of Dot Product

① $\langle 3,5\rangle\cdot\langle -1,2\rangle =$

$-3+10=\boxed{7}$

② $\langle 2,3\rangle\cdot\langle -3,2\rangle =$

③ $\langle 1,-3,4\rangle\cdot\langle 1,5,2\rangle =$

$(1\cdot 1)+(-3\cdot 5)+(4\cdot 2)$

$1 + -15 + 8$

$\boxed{-6}$

12:07

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Panel 14

Properties of Dot Product

a) $\vec{a}\cdot\vec{a} = \|a\|^2$

b) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

c) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$\vec{a}\cdot\vec{a} = \langle a_1,a_2\rangle\cdot\langle a_1,a_2\rangle = a_1a_1+a_2a_2 =$
$a_1^2+a_2^2 = \|\vec{a}\|^2$

proof of (b) in HW for the math gang.

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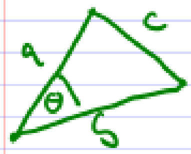
Panel 15

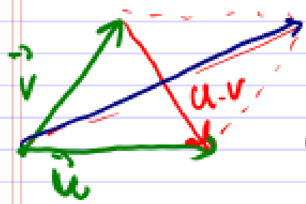
Theorem: If u and v are non-zero vectors in $\mathbb{R}^2$ then

$$\frac{u\cdot v}{\|u\|\cdot\|v\|} = \cos(\theta) \qquad (\text{Big deal})$$



law of cosine

$$c^2 = a^2+b^2-2ab\cos(\theta)$$



$$\|u-v\|^2 = \|u\|^2+\|v\|^2-2\|u\|\|v\|\cos(\theta)$$

$$(u-v)\cdot(u-v) = u\cdot u + v\cdot v - 2\|u\|\|v\|\cos(\theta)$$

$$\cancel{uu} - vu - uv + \cancel{vv} = \cancel{uu} + \cancel{vv} - 2\|u\|\|v\|\cos(\theta)$$

$$\cancel{-2}\,u\cdot v = \cancel{-2}\,\|u\|\|v\|\cos(\theta)$$

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Panel 16

Ex: Find angle between $u = i - 2j + 2k =$ and

a) $v = -3i + 6j + 2k$

don't care for angle. Go for $\cos(\theta)$

$$\cos(\theta) = \frac{\langle 1,-2,2\rangle\cdot\langle -3,6,2\rangle}{\|\langle 1,-2,2\rangle\|\cdot\|\langle -3,6,2\rangle\|} = \frac{-3-12+4}{3\sqrt{49}} = -11/21$$

b) $w = 2i + 7j + 6\vec{k}$

$$\cos(\theta) = \frac{\langle 1,-2,2\rangle\cdot(2,7,6)}{\|\quad\|\cdot\|\quad\|} = \frac{2-14+12}{\#} = \frac{0}{\#} = 0$$

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Panel 17

Corollary: Two vectors $\vec{v}$ and $\vec{w}$ are perpendicular iff $\vec{v}\cdot\vec{w} = 0$

Ex: Which of the following vectors are perpendicular?

a) $\langle 1,2,3\rangle$ and $\langle -1,-2,-3\rangle$

b) $\langle 1,2,3\rangle$ and $\langle -1,-3,2\rangle$

c) $\langle 1,2,3\rangle$ and $\langle 6,-1,1\rangle$

d) $\langle 1,2,3\rangle$ and $\langle 5,-1,1\rangle$

e) $\langle 1,2,3\rangle$ and $\langle 0,-3,2\rangle$

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Panel 18

Ex: Find the angle that $\vec{a} = \langle 1,2,3\rangle$ makes with the y-axis:

$\vec{i} = \langle 1,0,0\rangle$ (x-axis)

$$\cos(\theta) = \frac{2}{\sqrt{14}} = \frac{\overset{\langle 0,1,0\rangle}{\vec{j}} \cdot \langle 1,2,3\rangle}{\|\vec{j}\|\sqrt{14}}$$

$\vec{j} = \langle 0,1,0\rangle$

$\vec{k} = \langle 0,0,1\rangle$ z-ax

Directional Angles of $\vec{v} = \langle v_1, v_2, v_3\rangle$

with x-axis: $\cos(\theta_x) = v_1/\|v\|$ $\qquad \dfrac{\vec{i}\cdot\langle v_1,v_2,v_3\rangle}{\|\vec{i}\|\,\|v\|} = \dfrac{v_1}{\|v\|}$

y-axis: $\cos(\theta_y) = v_2/\|v\|$

z-axis: $\cos(\theta_z) = v_3/\|v\|$