Panel 1

Last time

Review of Calc 1 + 2
- limits
- cont.
- deriv
- integration

} one variable

$f(x,y) = x^2 + y^2$

$r(t) = \langle t^2, t^3 + t \rangle$

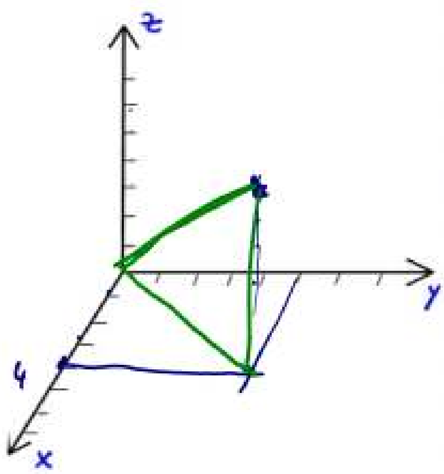
$v(x,y) = \langle x^2 y^2, x - y \rangle$

} calc 3

1

Panel 2

Plot the following Points.



P(3,2,1) - use blue

Q(4,5,6) - use green

I = (1,0,0)

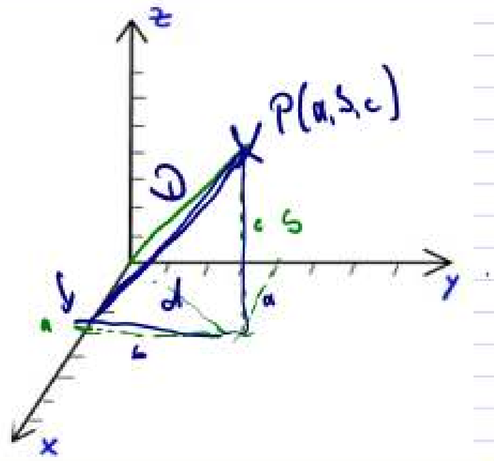
J = (0,1,0)

K = (0,0,1)

2

Panel 3

Distance in R^3



$P(a, b, c)$

Distance between origin and P

$d = \sqrt{a^2 + b^2}$

$D = \sqrt{d^2 + c^2} = \sqrt{a^2 + b^2 + c^2}$

Distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

3

Panel 4

$P(3,2,1)$ and $Q(4,5,6)$

Find distance to origin and distance P to Q

D of P to O: $\sqrt{3^2+2^2+1^2} = \sqrt{14}$

Dist. of Q to O: $\sqrt{4^2+5^2+6^2} = \sqrt{\#}$

$\text{dist}(P,Q) = \sqrt{(4-3)^2 + (5-2)^2 + (6-1)^2} = \sqrt{1+9+25} = \sqrt{35}$

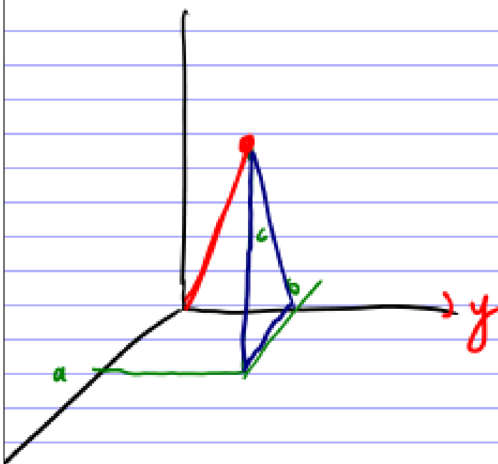
Dist. of P to xy-plane? To x-axis?

$\text{dist}(P, xy\text{-plane}) = 1$

$\text{dist}(P, x\text{-axis}) = \sqrt{2^2+1^2} = \sqrt{5}$

4

Panel 5



$d($ $, x\text{-axis}) = \sqrt{b^2+c^2}$

$d((a,b,c), y\text{-axis}) = \sqrt{b^2+c^2}$

$d($ $z\text{-axis}) = \sqrt{a^2+b^2}$

5

Panel 6

3D Objects

$P(x,y,z) \Rightarrow d = \sqrt{x^2+y^2+z^2} \Rightarrow d^2 = x^2+y^2+z^2$

All points that are a distance of $d = \sqrt{x^2+y^2+z^2}$ away from origin forms a sphere, center $(0,0,0)$, radius d.

Def: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = r^2$ is sphere centered at (x_0,y_0,z_0) and radius r

Ex: $x^2+y^2+z^2-2x-4y+8z+17=0$

$x^2-2x+1 + y^2-4y+4 + z^2+8z+16 = -17+1+4+16$

$(x-1)^2 + (y-2)^2 + (z+4)^2 = 4$ Sphere, center at $(1,2,-4)$, $r=2$

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Panel 7

Def: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$ is disk centered at $P(x_0,y_0,z_0)$ with radius r.

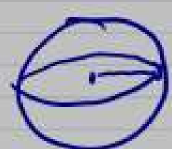
Ex: Find the center + radius of the sphere

$$x^2+y^2+z^2+10x+4y+2z-19=0$$

$$x^2+10x+25+y^2+4y+4+z^2+2z+1=19+25+4+1$$

$$(x+5)^2+(y+2)^2+(z+1)^2=49$$

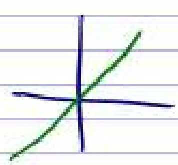
Sphere centered at $(-5,-2,-1)$, radius 7

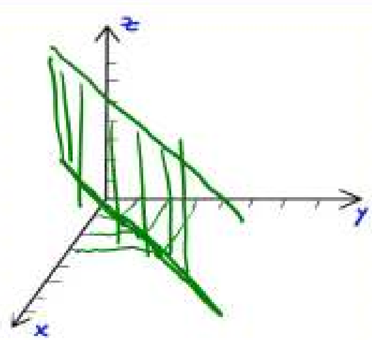


Panel 8

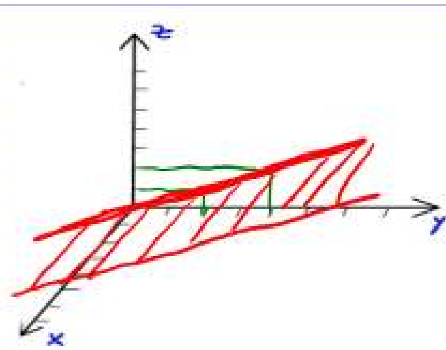
3D Objects

a) $y=x$





b) $z=\frac{1}{2}y$



Panel 9

Drawing 3D objects with Maple

Maple can easily draw 3D objects

Start Maple.

steel (one was. winning)

```
> with(plots);
> implicitplot3d(z=y^2, x=-3..3, y=-3..3, z=-1..9);
> plot3d(x^2, x=-3..3, y=-3..3);
> implicitplot3d(z^2+y^2=4, x=-3..3, y=-3..3, z=-3..3);
```

tube in yz plane around x-axis

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Panel 10

Ex: Use Maple to graph the following:

$x^2+y^2+z^2=4$ sphere

radius $\sqrt{2}$

$y^2+z^2=2$ tube in yz around x-axis.

$z=\sin(x)\cdot\cos(y)$

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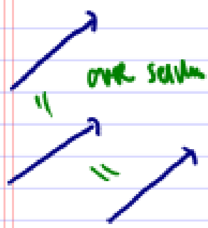
Panel 11

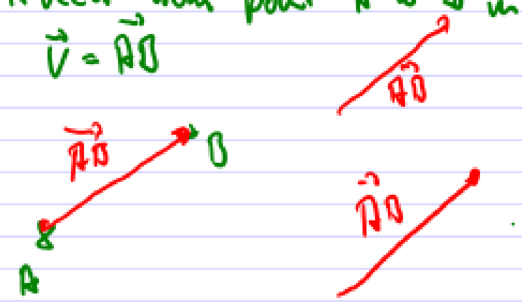
Vectors

Understand points in 3D (and 2D). Want to investigate more general objects ⇒ vectors.

Def: A vector is a directed line segment, i.e. a part of a line with a length and a direction (but no particular position)



A vector from point A to B is
$\vec{v} = \vec{AB}$



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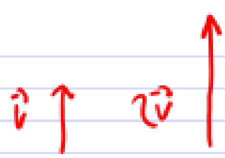
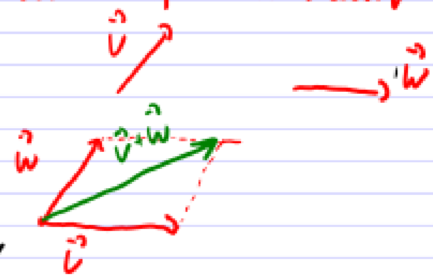
Panel 12

Vector Math, Geometrically



If $\vec{v}$ is a vector then

$k\cdot\vec{v}$, $k \in \mathbb{R}$ is same direction, k-times length.



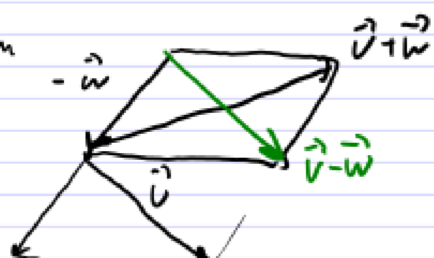
If $\vec{v}, \vec{w}$ are vectors, then

$\vec{v}+\vec{w}$ is main diag.

of parallelogram $\vec{v}, \vec{w}$

If $\vec{v}, \vec{w}$ are vectors, then

$\vec{v}-\vec{w}$ is



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Panel 13

Vector Math, Algebraically

practice

Algebraically v is described by components:

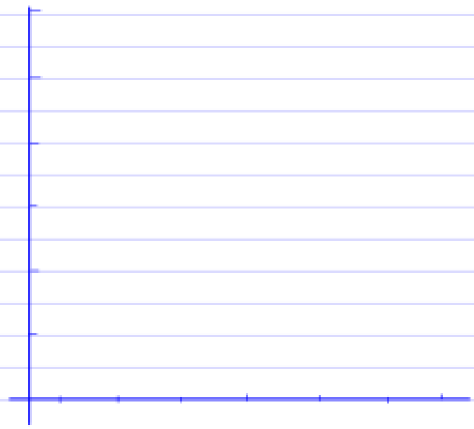
$\vec{v} = \langle v_1, v_2 \rangle$ or $\vec{v} = \langle v_1, v_2, v_3 \rangle$

Ex: Suppose $\vec{v} = \langle 1, 2 \rangle$, $\vec{w} = \langle 3, 1 \rangle$. Find

$\vec{v} + \vec{w}$

$\vec{v} + 2\vec{w}$

$3\vec{v} - \vec{w}$



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Panel 14

Vectors: Some Definitions

Def: The Length or norm of a vector $\vec{v} = \langle v_1, v_2 \rangle$ is: $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$

Def: A unit vector $\vec{v}$ is a vector such that $\|\vec{v}\| = 1$

Note: If $\vec{v} = \langle v_1, v_2 \rangle$ is any non-zero vector, then $\vec{u} = \frac{1}{\|v\|} \langle v_1, v_2 \rangle$ $\Rightarrow \|\vec{u}\| = \left\| \frac{1}{\|v\|} \langle v_1, v_2 \rangle \right\|$ is a unit vector pointing in the same direction as $\vec{v}$. $= \frac{1}{\|v\|} \|\langle v_1, v_2 \rangle\| = 1$

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Panel 15

Ex: unit vector

a) $\langle \frac{1}{2}, \frac{3}{4} \rangle$ $\quad \left\| \langle \frac{1}{2}, \frac{3}{4} \rangle \right\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2} \neq 1$

b) $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

Ex: Find unit vector in the direction of $\vec{v} = \langle 1, -5 \rangle$ and $\vec{w} = \langle 3, 2, -1 \rangle$

$$\|\vec{w}\| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1 \checkmark$$

Turn $\langle 1, -5 \rangle$ into unit vector

$\frac{1}{\sqrt{26}} \langle 1, -5 \rangle$ has length 1

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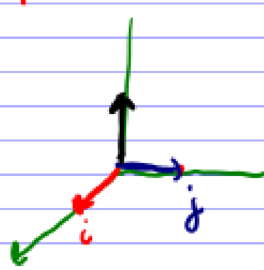
Panel 16

Special vectors in 3D

$\vec{i} = \langle 1, 0, 0 \rangle$ (x-axis)

$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$



are called basic unit vectors.

Every other vector can be written as a linear combination of basic unit vectors:

book   my notation

$$\vec{v} = -2\vec{i} + 3\vec{k} = \langle -2, 0, 3 \rangle$$

$$= -2\langle 1,0,0 \rangle + 3\langle 0,0,1 \rangle = \langle -2, 0, 0 \rangle + \langle 0, 0, 3 \rangle = \langle -2, 0, 3 \rangle$$

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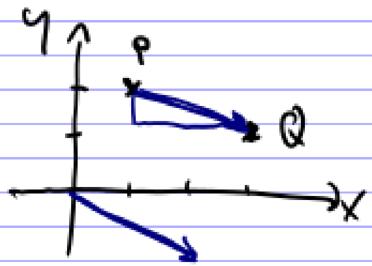
Panel 17

Vectors and Points

P (1,2) is a point, $\vec{v} = \langle 1,2 \rangle$ is a vector

Find vector from P(1,2) to Q(3,1)



$$\vec{PQ} = \langle 3-1, 1-2 \rangle = \langle 2,-1 \rangle$$

Thus if $P(x_1, y_1, z_1)$
$Q(x_2, y_2, z_2)$

then $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$
is vector from P to Q.

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Panel 18

Know how to add/subtract vectors.

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