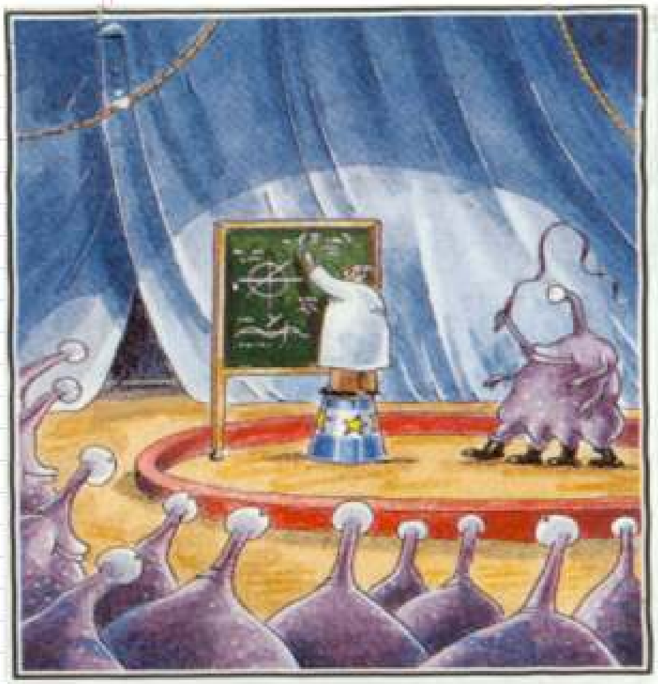
Panel 1



Abducted by an alien circus company, Professor ~~Doyle is~~ forced to write calculus equations in center ring.

Math 2411 / 2511

Calculus 3

Welcome!

Bert Wachsmuth

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Panel 2

About this Course:

http://pirate.shu.edu/~wachsmut/

DyKnow: login: username (8 letters)
password: same (not email pwd)

Syllabus: 3 exams:

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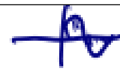
Panel 3

Calc 1 – Overview

$f(x)=x^3$ 

	Def.	Geometry	How-to
• Limit $\lim_{x\to c} f(x)=L$	Given any $\varepsilon>0$ there is a $\delta>0$ s.t. $\lvert f(x)-L\rvert<\varepsilon$ if $\lvert x-c\rvert<\delta$	as x gets close to c, $f(x)$ gets close to L	plug in c, hope it works or use l'Hospital
• Continuity	$\lim_{x\to c} f(x)=f(c)$ ① = ②	no gaps, no holes	check the limit.
• Differentiation	$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$	slope of tangent (no kinks, no corners)	power rule, quotient, product, chain
• Integration $\int_a^b f(x)\,dx$	$\lim_{n\to\infty}\sum^n f(x_i)\Delta x_i$, limit of Riemann sums	area under curve, if $f\geq 0$	$\int_a^b f(x)\,dx=F(b)-F(a)$, subst.

Panel 4

Calc 1: Some Refresher

① $\lim_{x\to 1}\frac{x^2-1}{x}=\frac{0}{1}=0$ $\quad\lim_{x\to 2}\frac{x^2-4}{x-2}=\frac{0}{0}=\lim_{x\to 2}\frac{2x}{1}=4$ $\quad\lim_{x\to 0}\frac{x^2\sin(x)}{1-\cos(x)}=\frac{0}{0}$ $=\lim_{x\to 0}\frac{2x\sin(x)+x^2\cos(x)}{\sin(x)}=\frac{0}{0}=$ again

② $f(x)=x^2-\sin(x)+e^x-\sqrt{x}+\ln(x)$

$f'(x)=2x-\cos(x)+e^x-\frac{1}{2}x^{-1/2}+\frac{1}{x}$

$g(x)=\frac{x\sin(x)}{\sqrt{1-x^2}}$

$$g'(x)=\frac{(\sin(x)+x\cos(x))\sqrt{1-x^2}-x\sin(x)\cdot\left(\frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)\right)}{\left(\sqrt{1-x^2}\right)^2}$$

③ $\int 5x^1-3\sqrt{x}+e^x-\frac{5}{x}-\frac{6}{x^2}\,dx$

$=5\frac{1}{2}x^2-3\frac{2}{3}x^{3/2}+e^x-5\ln(x)-6(-1)x^{-1}+C$

Panel 5

Calc 2 - Overview

- Heavy Duty Integration Techniques
  eg: int parts, PFD, inverse trig
- Applications of Integration
  e.g.: ~~center of mass, inertia~~, revolutions
- Sequences & Series
  eg: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- Differential Equations
  e.g. $y' = y \Rightarrow y = ke^t$

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Panel 6

Calc 2 Refresher

$= -x\cos(x) + \int \cos(x)\,dx$

~~$\int \frac{d}{dx}$~~ $(uv) = \int u'v + \int uv'$

① $\int x \sin(x)\,dx = \frac{1}{2}x^2\sin(x) - \int \frac{1}{2}x^2\cos(x)\,dx$ $\int u'v = uv - \int uv'$

$u' = x \Rightarrow u = \frac{1}{2}x^2$, $v = \sin(x) \Rightarrow v' = \cos(x)$ | $u' = \sin(x) \Rightarrow u = -\cos(x)$, $v = x \Rightarrow v' = 1$ ✓

② $\int \frac{5x}{x^2 - x - 2}\,dx = \int \frac{10/3}{x-2} + \frac{5/3}{x+1}\,dx = \frac{10}{3}\ln|x-2| + \frac{5}{3}\ln|x+1| + C$

$\frac{5x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{A(x+1) + B(x-2)}{(x-2)(x+1)}$

$A(x+1) + B(x-2) = 5x \;\forall x$

$x = 2: 3A = 10 \Rightarrow A = 10/3$

$x = -1: -3B = -5 \Rightarrow B = 5/3$

③ $\frac{5x}{1+x^2}$ as a power series

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Panel 7

$$\frac{5x}{1+x^2} = 5x \cdot \frac{1}{1-(-x^2)} = 5x\left(1+(-x^2)+(-x^2)^2+(-x^2)^3+\dots\right)$$

$$= 5x - 5x^3 + 5x^5 - 5x^7 + 5x^9 - 5x^{11} + 5x^{13} - \dots$$

$\frac{1}{2} \; \frac{1}{4} \; \frac{1}{8} \; \frac{1}{16} \; \frac{1}{32} \; \frac{1}{64}$

Hint: $\frac{1}{1-x} = 1 + x + x^2 + \dots$

Ex: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots = \frac{1}{1-\frac{1}{2}} = 2$

$$1 + x + x^2 + x^3 + \dots + x^n = S$$

$$-\left(x + x^2 + x^3 + x^4 + \dots + x^{n+1}\right) = xS$$

$$1 - x^{n+1} = S - xS = S(1-x)$$

$$\frac{1-x^{n+1}}{1-x} = S = 1 + x + \dots + x^n$$

$$\lim_{n\to\infty} \frac{1-x^{n+1}}{1-x} = \frac{1}{1-x} \quad \text{if } x<1,\ x^{n+1} \to 0$$

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Panel 8

Calc 3

Calc 1 + Calc 2 in several variables

$f: \mathbb{R} \to \mathbb{R}$, e.g. $f(x) = x^2$ calc 1

$f: \mathbb{R}^2 \to \mathbb{R}$, e.g. $f(x,y) = x^2 + y^2$

$f: \mathbb{R} \to \mathbb{R}^2$, e.g. $f(t) = \langle \cos(t), \sin(t) \rangle$

$f: \mathbb{R}^2 \to \mathbb{R}^2$, e.g. $f(x,y) = \langle x^2 + y^2, 2xy \rangle$

$f: \mathbb{R}^3 \to \mathbb{R}$, e.g. $f(x,y,z) = x^2 + y^3 + z^4$

$f: \mathbb{R}^5 \to \mathbb{R}$, e.g. $f(x_1,x_2,x_3,x_4,x_5) = x_1 \cdot x_2^2 + x_3 - x_4^2 x_5$

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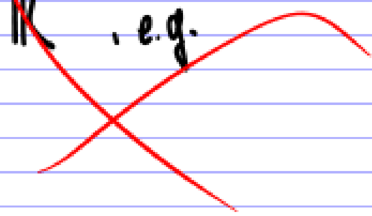
Panel 9

$f: \mathbb{R} \to \mathbb{R}$, e.g.



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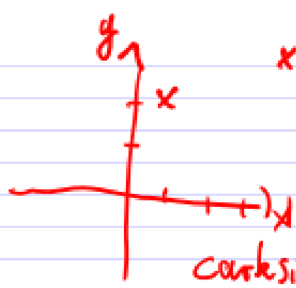
Panel 10

Introducing $\mathbb{R}^3$

Coordinate system in $\mathbb{R}^2$:

Ex: plot $(1,2)$, $(4,3)$

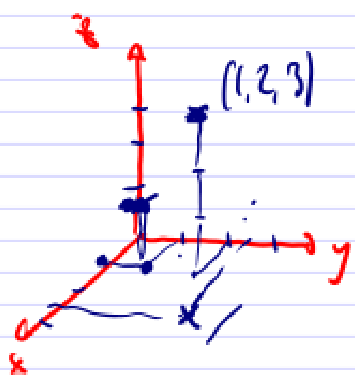


cartesian coord.

R. Descartes

Coordinate system in $\mathbb{R}^3$:

Ex: plot $(1,1,1)$, $(1,2,3)$



x-axis: all points $(t, 0, 0)$

y-axis: $(0, t, 0)$

z-axis: $(0, 0, t)$

xy plane: all points $(x, y, 0)$

yz plane: $(0, y, z)$

xz plane: $(x, 0, z)$

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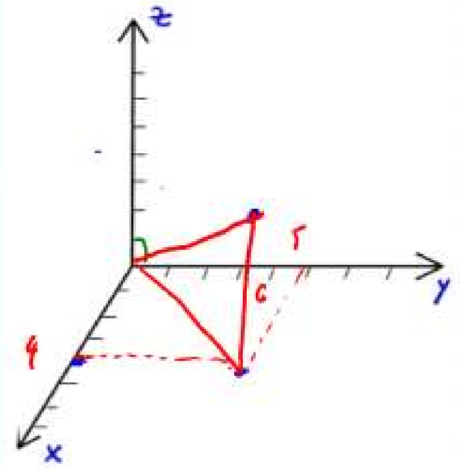
Panel 11

Ex: Plot the following Points.



P(3,2,1) - use blue

Q(4,5,6) - use green

I=(1,0,0)

J=(0,1,0)

K=(0,0,1)

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Panel 12

Given $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ if $|x - c| < \delta$.

stacey Ballon

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