Calc I: Assignment 24

① Consider the lamina D bounded by

$x = 1 - y^2$ and the coordinate axis in the 1st

quadrant with density function $\rho(x,y) = y$

Find the mass of the lamina and the center

of mass. Illustrate.

② Sketch the following vector fields

a) $\vec{F}(x,y) = \langle 1, x \rangle$

b) $\vec{F}(x,y) = \langle y, \frac{1}{2} \rangle$

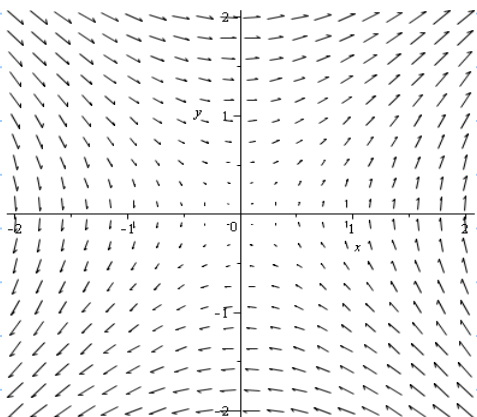c) $\vec{F}(x,y) = \frac{1}{\sqrt{x^2+y^2}} \langle y, x \rangle$

③ Match the vector fields with the plots.
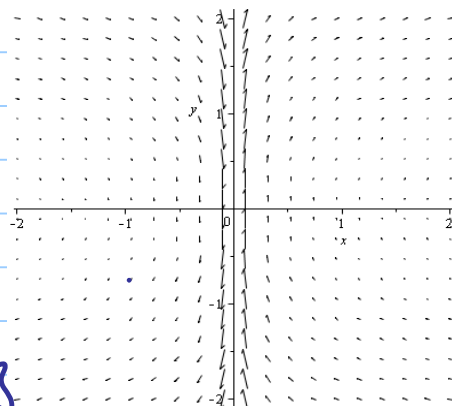
a) $F(x,y) = \langle y, \frac{1}{x} \rangle$ b) $F(x,y) = \langle x-2, x+1 \rangle$
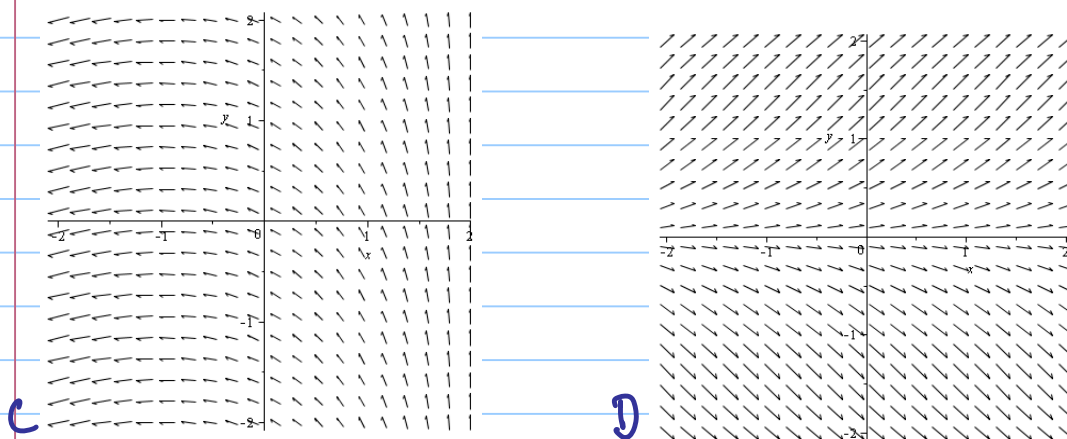
c) $F(x,y) = \langle y, x \rangle$ d) $F(x,y) = \langle 1, \sin(y) \rangle$

A   B

C

D

④ Use Maple to plot $\vec{F}(x,y) = \langle y^2 - 2xy, 3xy - 6x^2 \rangle$

⑤ Recall that if $\vec{F} = \langle M, N \rangle$ is conservative, then $\dfrac{\partial N}{\partial x} = \dfrac{\partial M}{\partial y}$. Which vector fields are conservative:

a) $\vec{F} = \langle 2x - 3y, -3x + 4y - 9 \rangle$

b) $\vec{F} = \langle e^x \cos(y), e^x \sin(y) \rangle$

c) $\vec{F} = \langle 3x^2 + 2y^2, 4xy + 3 \rangle$

⑥ For the vector fields in ⑤ that are conservative, find the potential function

⑦ Find the curl (F) and div (F) for:

a) $\vec{F} = \langle xyz, 0, -x^2 y \rangle$

b) $\vec{F} = \langle x^2 yz, xy^2 z, xy z^2 \rangle$

c) $\vec{F} = \langle e^x, e^{xy}, e^{xyz} \rangle$

① If $f: \mathbb{R}^3 \to \mathbb{R}$ is a function and $\vec{F}: \mathbb{R}^3 \to \mathbb{R}^3$
is a vector field, which expression is meaningful:

curl (f)

grad (f)

div ($\vec{F}$)

curl (grad (f))

grad ($\vec{F}$)

grad (div ($\vec{F}$))

div (grad (f))

grad (div (f))

curl (curl ($\vec{F}$))

div (div ($\vec{F}$))

(grad (f)) × (div ($\vec{F}$))

div (curl (grad (f)))

② Which of there vector fields is conservative:

a) $\vec{F} = \langle y^2 z^3, 2xy z^3, 3xy^2 z^2 \rangle$

s) $\vec{F} = \langle e^z, 1, xe^z \rangle$

c) $\vec{F} = \langle y \cos(xy), x \cos(xy), -\sin(z) \rangle$