Calc 3 - Assignment # 19

① Find the absolute max and min for

a) $f(x,y) = x^2 + y^2 + x^2 y + 4$ in $[-1,1] \times [-1,1]$

b) $f(x,y) = 3 + xy - x - 2y$ in the triangular region with vertices $(1,0)$, $(5,0)$, and $(1,4)$

② Use the method of Lagrange multipliers to find the extreme values of $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

③ Estimate the volume below $z = xy$ and above the rectangle $D = [0,6] \times [0,4]$ by dividing the x-interval into 4 points, the y-interval into 3 points, and taking as height the value of $f(x,y)$ at each upper-right corner. Compare your answer with

$$\iint_D xy \, dA$$

④ Evaluate $\iint_R 5 - x \, dA$, $R = [0,5] \times [0,3]$

both algebraically and geometrically.

(5) Use Fubini's Theorem to compute:

a) $\int_0^3 \int_0^1 1+4xy \, dx \, dy$

b) $\int_0^1 \int_1^2 4x^3 - 9x^2y^2 \, dy \, dx$

c) $\int_0^1 \int_0^1 xy\sqrt{x^2+y^2} \, dy \, dx$

d) $\int_0^1 \int_0^1 \sqrt{s+t} \, ds \, dt$

e) $\iint_R \frac{1+x^2}{1+y^2} \, dA$, $R = [0,1] \times [0,1]$

f) $\iint_R \frac{x}{x^2+y^2} \, dA$, $R = [1,2] \times [0,1]$

(6) Find the volume under $z = 4 + x^2 - y^2$ and above $R = [-1,1] \times [0,2]$

(7) Find the volume of the solid enclosed by $z = 1 + e^x \sin(y)$ and the planes $x = \pm 1$, $y = 0$, $y = \pi$, and $z = 0$.

(8) Read Fubini's Theorem carefully. Then use Maple to compute

$$\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\, dy\, dx \quad \text{and}$$

$$\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\, dx\, dy.$$

Explain.