

# Calc 3 - Assignment 16

Note Title

10/14/2011

① Find the directional derivative of  $f(x,y)$  at the given point in the direction indicated by  $\theta$ :

a)  $f(x,y) = x^2 y^3 - y^4$ ,  $P(2,1)$ ,  $\theta = \pi/4$

b)  $f(x,y) = x \sin(xy)$ ,  $P(2,0)$ ,  $\theta = \pi/3$

② Find the gradient of  $f$  at the point  $P$ :

a)  $f(x,y) = \frac{y^2}{x}$ ,  $P(1,2)$

b)  $f(x,y,z) = \sqrt{x+yz}$ ,  $P(1,3,1)$

③ Find the directional derivative  $D_{\vec{v}}(f)$ :

a)  $f(x,y) = \ln(x^2 + y^2)$ ,  $\vec{v} = \langle -1, 2 \rangle$

b)  $f(x,y,z) = (x+2y+3z)^{3/2}$ ,  $\vec{v} = 2\vec{j} - \vec{k}$

④ Find the directional derivative of

$f(x,y,z) = xy + yz + zx$  at  $P(1,-1,3)$  in the direction of  $Q(2,4,5)$ .

⑤ Find max. rate of change and its direction:

a)  $f(x,y) = \frac{y^2}{x}$  at  $(2,4)$

b)  $f(x,y,z) = \frac{x+y}{z}$  at  $(1,1,-1)$

⑥ The depth of a lake is given by  
 $z = 200 + 0.02x^2 - 0.001y^3$  measured in feet.

A fisherman in a small boat starts at  $(80, 60)$  and moves toward  $(0, 0)$ . Is the water under the boat getting deeper or shallower?

⑦ The temperature at a point  $(x, y, z)$  is  
 $T(x, y, z) = 200 e^{-x^2 - 3y^2 - 9z^2}$

a) Find the rate of change of  $T$  at  $P(2, -1, 2)$   
in the direction toward  $(3, -3, 3)$

b) In which direction does  $T$  increase  
the fastest?

c) Find the max. rate of increase at  $P$

optional

⑧ A function is called homogeneous of degree  
 $n$  if  $f(tx, ty) = t^n f(x, y)$  for all  $t$

a) Show that  $f(x, y) = x^2y + 2xy^2 + 5y^3$  is  
homogeneous of degree 3

g) Show that if  $f$  is homogeneous of degree  $n$ , then

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = n \cdot f(x, y)$$

Hint: Use chain rule to differentiate  $f(tx, ty)$  with respect to  $t$ .