Panel 1

Integration Summary

contour or line integrals

- function
  - $\int f(x,y)\,ds$
  - $\int f(x,y)\,dx$
  - $\int f\,dy$
- vector field: $\int \vec{F}\,d\vec{r}$
  - $\int M\,dx + N\,dy + P\,dz$
  - $\xrightarrow{\text{conservative}}$ $f(b) - f(a)$
  - closed curve, 2D only: $\oint_C M\,dx + N\,dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dA$ — Green's Thm

1

Panel 2

② Evaluate the following integrals:

a) $\oint_\gamma (xy^2 + e^y)\,dx + (x^2y + xe^y)\,dy$, γ = unit circle

b) $\int_{(-1,0)}^{(0,1)} (3x^2 + 2y)\,dx + (2x - 2y)\,dy$

c) $\int_C 3x^2 - 7yx\,ds$, C line from $(0,1)$ to $(2,3)$

d) $\int_\gamma \langle y, z, x\rangle \cdot d\vec{r}$, γ line from $(1,1,1)$ to $(2,3,4)$

e) $\oint \langle y, z, x\rangle\,d\vec{r}$, γ circle of radius 3

2

Panel 3

Alternate version of Green's Theorem:

If $\vec{F} = \langle M, N \rangle$, consider $\vec{F} = \langle M, N, 0 \rangle$

$\Rightarrow \mathrm{curl}(\vec{F}) = \langle 0, 0, \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$

$$\oint_C \vec{F}\, d\vec{r} = \iint_D \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA = \iint \mathrm{curl}(\vec{F}) \cdot \vec{k}\, dA$$

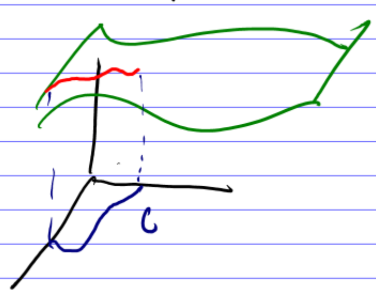
3

Panel 4

$y = f(x)$ a curve $(r(t) = \langle t, f(t) \rangle)$

$\int_C ds = \int \sqrt{1 + f'(x)^2}\, dx$ length of C

$\int_C g(x,y)\, ds = \int_a^b g(x, f(x)) \sqrt{1 + f'^2}\, dx$

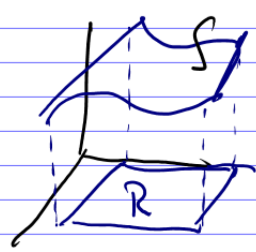


$z = f(x,y)$ a surface

$\iint_R dS := \iint_R \sqrt{1 + f_x^2 + f_y^2}\, dA$ surface area

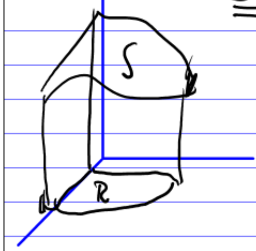
$\iint_S g(x,y,z)\, dS :=$ surface integral

$\iint_R g(x, y, f(x,y)) \sqrt{1 + f_x^2 + f_y^2}\, dA$



4

Panel 5



Def: Suppose surface S is defined by $z = f(x,y)$, $(x,y) \in R$. Then

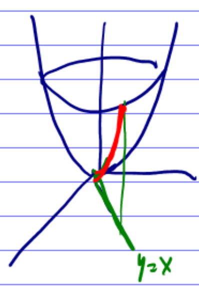
$$\iint_S g(x,y,z)\,dS = \iint_R g(x,y,f(x,y))\sqrt{1+f_x^2+f_y^2}\,dA$$

$\iint_S g(x,y,z)\,dS$ is the integral of that portion of $w = g(x,y,z)$ over surface S where S is a surface defined on R

5

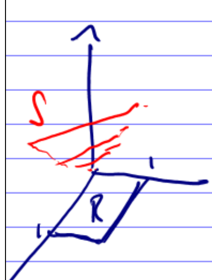
Panel 6

Ex: $\int_\gamma x^2+y^2\,ds$, $y = x$, $x \in [0,1]$



$$= \int_0^1 x^2+x^2\sqrt{1+1}\,dt = 2\sqrt{2}\,\frac{2}{3}$$

$\iint_S x^2+y^2+z^2\,dS$, $\underbrace{z = x+y}_{}$ $f(x,y)$, $x,y \in [0,1]\times[0,1]$



$$\iint_R x^2+y^2+(x+y)^2\sqrt{1+f_x^2+f_y^2}\,dA =$$

$$\int_0^1\int_0^1 \left(x^2+y^2+x^2+2xy+y^2\right)\sqrt{1+1+1}\,dy\,dx$$

6

Panel 7

Ex: Evaluate $\iint_S x^2 z \, dS$, S portion of $z^2 = x^2 + y^2$ between $z = 1$ and $z = 4$.

$z = \sqrt{x^2+y^2}$

$$\iint_{S \in \mathbb{R}^3} x^2 z \, dS = \iint_{R \in \mathbb{R}^2} \_\_ \sqrt{\quad} \, dA$$

$$\iint_S x^2 z \, dS = \iint_R x^2 \sqrt{x^2+y^2} \sqrt{\cancel{1+f_x^2+f_y^2}^{\,2}} \, dA = \iint_R x^2 \sqrt{x^2+y^2} \sqrt{2} \, dA =$$

$f_x = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$, $f_y = \frac{y}{\sqrt{x^2+y^2}}$

$1 + f_x^2 + f_y^2 = 1 + \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} = 2$

$$= \int_0^{2\pi} \int_1^4 r^2 \cos^2(t) \, r \sqrt{2} \, r \, dr \, dt$$

7

Panel 8

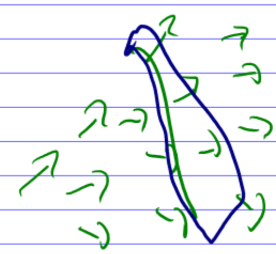
Surface integrals have functions as integrand. Want to extend that integral to vector fields:

Def: If S is surface given by $z = f(x,y)$, over region R.

$\Rightarrow \iint_S \vec{F} \cdot \vec{n} \, dS$ is the flux of a vector field

where $\vec{n}$ is the normal vector to the surface S

given by



8