Panel 1

Applications of Integration

① $\iint_R f(x,y)\,dA$ = volume of $z = f(x,y)$ over R if $f \geq 0$

$\iint_R 1\,dA$ = area(R)

② Mass of a Lamina ← thin plate

Suppose we have a lamina with density function $\rho(x,y)$

if rectangle is small, ρ is const.

→ mass: $\rho(x_i, y_j)\,\Delta x_i\,\Delta y_j$

total Mass: $M = \sum_i \sum_j \rho(x_i, y_j)\,\Delta x_i\,\Delta y_j = \iint_R \rho\,dA$

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Panel 2

Lamina with Density function $\rho(x,y)$

Mass: $\iint_D \rho(x,y)\,dA$

Moments: $M_x = \iint_D y\,\rho(x,y)\,dA$ (about x-axis)

$M_y = \iint_D x\,\rho(x,y)\,dA$ (about y-axis)

Center of Gravity: $(\bar{x}, \bar{y})$ where

$\bar{x} = M_y / M$, $\bar{y} = M_x / M$

$M = \iint \rho(x,y)\,dA$ = total mass

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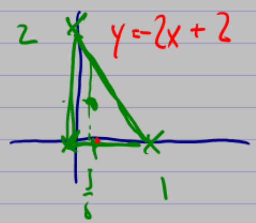
Panel 3

Ex. Find center of gravity for triangular lamina with vertices $(0,0)$, $(1,0)$, and $(2,0)$ if $\rho(x,y) = 1 + 3x + y$



$$M = \iint_D 1 + 3x + y \, dA = \int_0^1 \int_0^{-2x+2} 1 + 3x + y \, dy \, dx = 8/3$$

$$M_x = \iint y \rho(x,y) \, dA = \int_0^1 \int_0^{-2x+2} y(1 + 3x + y) \, dy \, dx = 11/6$$

$$M_y = \iint x \rho(x,y) \, dA = \int_0^1 \int_0^{-2x+2} x(1 + 3x + y) \, dy \, dx = 1$$

$$\Rightarrow (\bar{x}, \bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right) = \left(1/\tfrac{8}{3}, \tfrac{11}{6}/\tfrac{8}{3}\right) = \left(\frac{3}{8}, \frac{11}{6} \cdot \frac{3}{8}\right) = \left(\frac{3}{8}, \frac{11}{16}\right)$$

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