Panel 1

Last Time

Abs. extrema: critical pts inside D, on the boundary of D, and endpoints

Reading

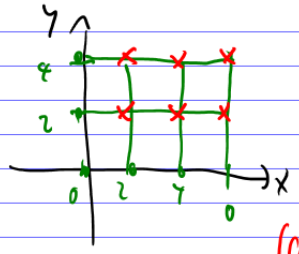
↓ Lagrange Multipliers: $\nabla f = \lambda \nabla g$, $g = k$ — max. f given $g = k$

Integration:

- Definition: $\iint_D f(x,y)\,dA = \lim_{\text{infmax}} \sum_{i,j} f(x_i, y_j)\,\Delta x_i\,\Delta y_j$
- Interpretation: is volume under $z = f(x,y)$, as long as $f \geq 0$
- How-to: Fubini

1

Panel 2



$z = f(x,y) = xy$

$\sum f(x_i, y_j)\,\Delta x_i\,\Delta y_j = \left(f(2,2) + f(4,2) + f(6,2) + f(2,4) + f(4,4) + f(6,4)\right) \cdot 2 \cdot 2 =$

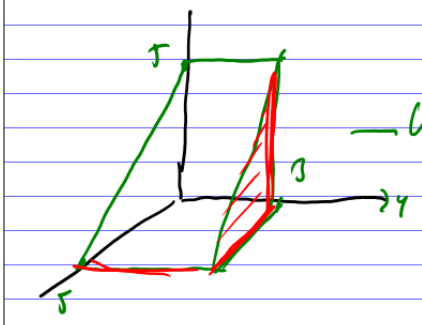
Compare to: $\iint_D f(x,y)\,dA = \int_0^4 \left(\int_0^6 xy\,dx \right) dy =$

$= \int_0^4 \frac{1}{2} x^2 y \Big|_{0=x}^{6} dy = \int_0^4 18y\,dy$

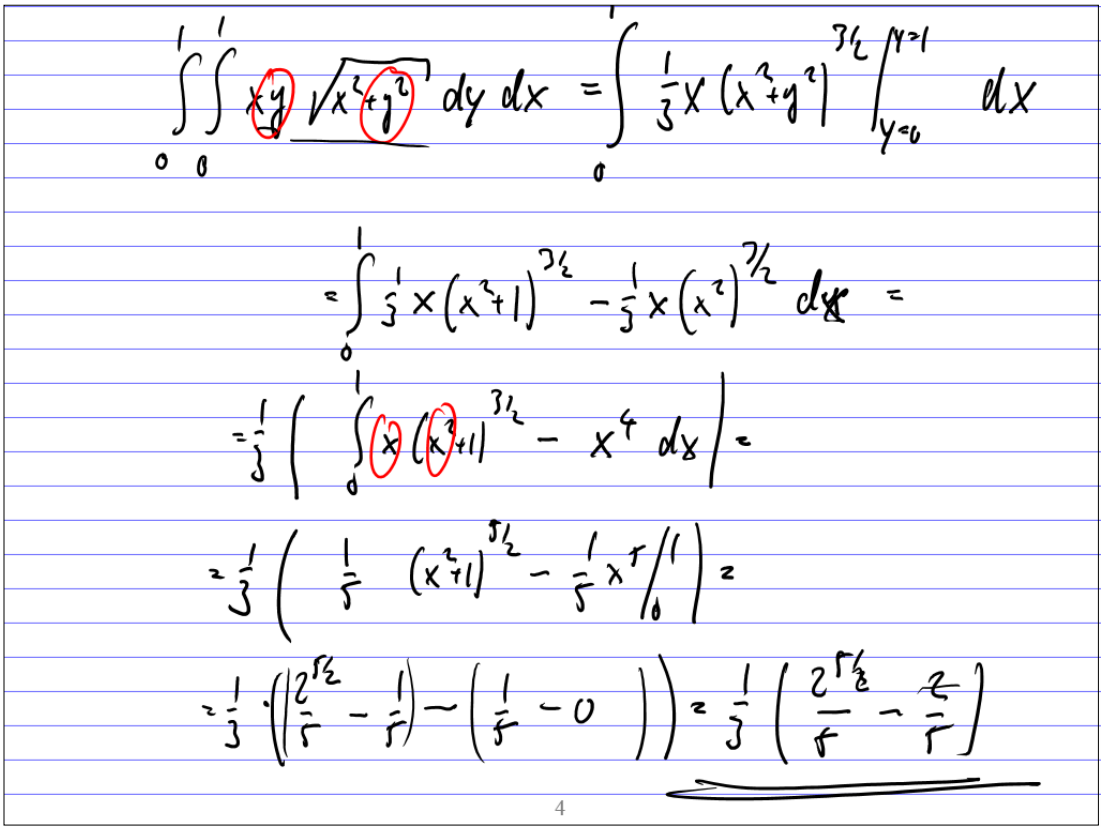
$= 9y^2 \Big|_0^4 = 9 \cdot 16$

2

Panel 3



Panel 4



Panel 5

Name: _____

Quiz:

① Evaluate the following integrals

a) $\int_0^2 \int_0^1 2x + 4xy \, dy \, dx$

b) $\iint_R 2x e^{x^2} + 2y \, dA \quad , \quad R = [0,1] \times [0,2]$

5

Panel 6

② Find the volume of the solid enclosed by the surface $z = x^2 y + xy^2$ and the planes $z = 0$, $x = 0$, $x = 1$, $y = 0$, and $y = 1$

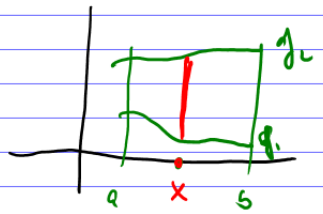
6

Panel 7

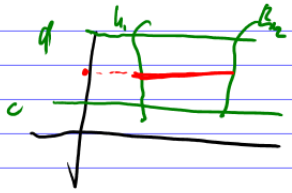
More on Integration

Type 1 and type 2 domain integration

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$$



$$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dx\, dy$$

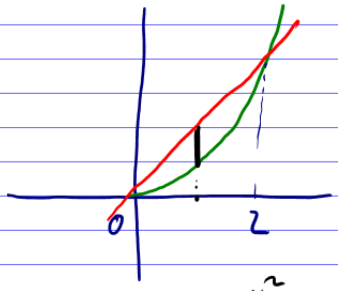


7

Panel 8

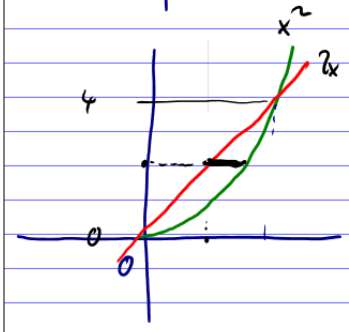
Ex: Volume under $z = x^2+y^2$ above $y=2x$ and $y=x^2$

$\frac{y}{2} = x$ $\sqrt{y} = x$



$$\int_0^2 \int_{x^2}^{2x} x^2+y^2\, dy\, dx =$$

Sum

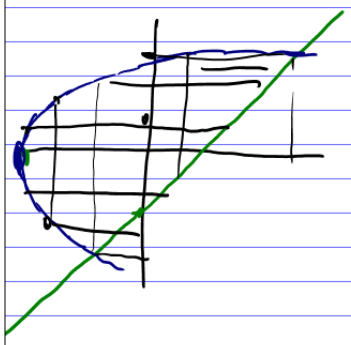


$$\int_0^4 \int_{y/2}^{\sqrt{y}} x^2+y^2\, dx\, dy =$$

8

Panel 9

Ex: Find $\iint_D xy\,dA$ where D is bounded by $y = x-1$ and $y^2 = 2x+6$. Should you $\iint xy\,dx\,dy$ or $\iint xy\,dy\,dx$?



$\iint xy\,dx\,dy$ one integral

$\iint xy\,dy\,dx$ must use 2 integrals

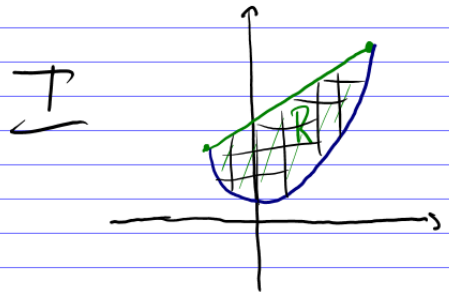
$y^2 = 2x+6$

$y^2 - 6 = 2x$, $x = \frac{1}{2}y^2 - 3$

9

Panel 10

$dx\,dy$ or $dy\,dx$?

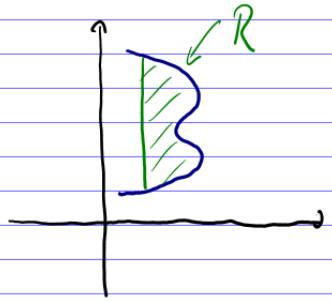


a) $\iint_R f(x,y)\,dx\,dy$ horizontal

b) $\iint_R f(x,y)\,dy\,dx$ vertical

① $\iint_R f(x,y)\,dx\,dy$ horiz.

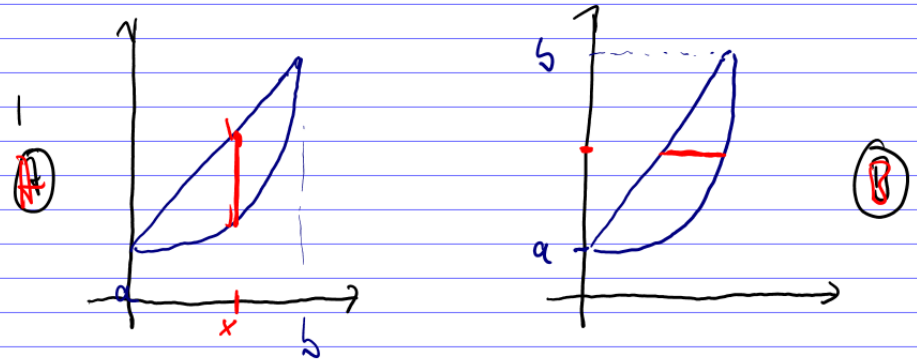
② $\iint_R f(x,y)\,dy\,dx$ vert.



10

Panel 11

Which picture represents $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$?

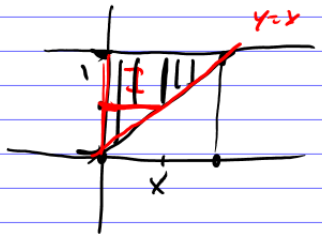


11

Panel 12

Occasionally there are other reasons that you would prefer one integration order over another - after all, you do have choices.

Ex: Find $\int_0^1 \int_{x=y}^{1} \sin(y^2)\, dy\, dx = \int_0^1 \int_0^{x=y} \sin(y^2)\, dx\, dy$



$= \int_0^1 x \sin(y^2) \Big|_{x=0}^{x=y} dy =$

$= \int_0^1 y \sin(y^2)\, dy = -\frac{1}{2}\cos(y^2)\Big|_0^1$

12

Panel 13

$$\int_0^2 \int_0^1 x^2 y \, dx \, dy$$

HW

$$\int_0^1 \int_0^y \sqrt{1-y^2} \, dx \, dy$$

$$\int_0^1 \int_{2x}^2 e^{y^2} dy \, dx$$

Exam 2 next week Friday
(or Wed)

13