Panel 1

Sheets + Coordinate Systems — alg. + geometrically

Vectors: add, subtract, length, unit vectors

dot + cross product, angles, projection

Planes + Lines: equations, intersect, distances

Vector-valued functions: limit, deriv, int., arc length

$\vec{T}, \vec{N}, \vec{B}$, curvature

Motion in Space: velocity, speed, acceleration,

normal + tang. comp. of acceleration, story problems

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Panel 2

Formulas:

$\cos(\theta) = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|}$

$\|\vec{u}\| = \sqrt{v_1^2+v_2^2+v_3^2}$

$v = r'(t)$

$\text{proj}_{\vec{v}}(\vec{w}) = \frac{\|v\cdot w\|}{\|\vec{v}\|^2}\vec{v}$

$s = \|\vec{v}\| = \|\vec{r}'\|$

$T = r'/\|r'\|$

$a = r''$

$a_T = \frac{\vec{v}\cdot\vec{a}}{s}$

$N = T'/\|T'\|$

$a_N = \frac{\|\vec{v}\times\vec{a}\|}{s}$

$B = T \times N$

$\kappa = \frac{\|r'\times r''\|}{\|r'\|^3}$

Supporting plane: B as normal vector

$s = \int_a^b \|r'(t)\|\,dt$

Osculating circle: $1/\kappa$ as radius

distance: $d = \frac{|ax_0+by_0+cz_0+d|}{\|\vec{n}\|}$

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Panel 3

$r(t) = \langle t^2, 5t, t^2 - 16t \rangle$ min. speed

$r'(t) = \langle 2t, 5, 2t - 16 \rangle$

$s = \|r'\| = \sqrt{4t^2 + 25 + (2t-16)^2} =$

$= \sqrt{4t^2 + 25 + 4t^2 - 64t + 256} =$

$f(t) = \sqrt{8t^2 - 64t + 281}$ ~ d^2 min $= 8t^2 - 64t + 281$

$f'(x) = \frac{1}{2}(\quad)^{-1/2} \cdot (16t - 64) = 0$, $16t - 64 = 0$,

$t = 4$

yields min speed

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Panel 4

$\langle 3t - t^3, 3t^2, 0 \rangle$

$v = (3 - 3t^2, 6t)$

$a = \langle -6t, 6 \rangle$

$s = \sqrt{(3-3t^2)^2 + 36t^2}$

$= \sqrt{9 - 18t^2 + 9t^4 + 36t^2} =$

$= \sqrt{9 + 18t^2 + 9t^4} =$

$= \sqrt{(3 + 3t^2)^2} = 3 + 3t^2 =$

$= 3(1+t^2)$

$a_T = \frac{v \cdot a}{s} = \frac{-18t + 18t^3 + 36t}{3(1+t^2)} =$

$= \frac{18t(1+t^2)}{3(1+t^2)} = 6t$

$a_N = \frac{\|v \times a\|}{s} = \frac{18(1+t^2)}{3(1+t^2)} = 6$

$$\begin{vmatrix} i & j & k \\ 3-3t^2 & 6t & 0 \\ -6t & 6 & 0 \end{vmatrix} = \langle 0, 0, 18 - 18t^2 + 36t^2 \rangle$$

$\langle 0, 0, 18 + 18t^2 \rangle$

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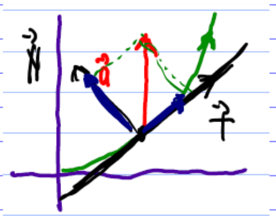
Panel 5



Find a_T and a_N

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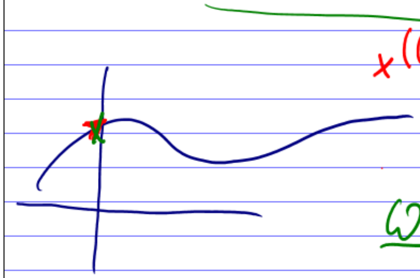
Panel 6

$\vec{r}(t) = \langle 3+t,\ 2+\ln(t),\ 7-\frac{4}{t^2+1}\rangle$ | coast to $(6,4,9)$



cut engines → coast along tangent line. $l(s)$.

Want: $l(s) = (6,4,9)$ for some s

$r'(t) = \langle 1, \frac{1}{t}, \frac{8t}{(t^2+1)^2}\rangle$: $l(s) = \langle 3+t, 2+\ln(t), 7-\frac{4}{t^2+1}\rangle + s\langle 1, \frac{1}{t}, \frac{8t}{(t^2+1)^2}\rangle$

$3+t+s = 6$, $s = 3-t$, $s = 2$

$2+\ln(t) + s/t = 4 \Rightarrow 2+\ln(t) + \frac{3-t}{t} = 4$, $t = 1$

$7 - \frac{4}{t^2+1} + s\frac{8t}{(t^2+1)^2} = 9$ $\quad 7 - \frac{4}{2} + \frac{16}{4} = 7-2+4 = 9$ ✓

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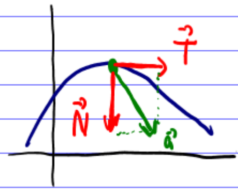
Panel 7



$$\vec{a} = a_t \cdot \vec{T} + a_N \vec{N}$$

Later

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Panel 8

$\vec{a} = a_t \vec{T} + a_N \vec{N}$ with

$a_T = s'$ and $a_N = s^2 \kappa$

<u>Later</u>

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Panel 9

True / False questions

$r(t) = \langle t^3, 2t^3, 3t^3 \rangle$ is a line

$\frac{d}{dt} \left(v(t) \times w(t) \right) = v'(t) \times w'(t)$

$\frac{d}{dt} \| r(t) \| = \| r'(t) \|$

If $\| r(t) \| = 1$ for all t then $r(t)$ is constant

If $\| r(t) \| = 5$ for all t then $r(t) \cdot r'(t) = 0$

More about distances, planes, and vectors

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Panel 10

Vectors: Suppose $u = \langle 7, -2, 3 \rangle$, $v = \langle -1, 4, 5 \rangle$, and $w = \langle -2, 1, -3 \rangle$

Are u and v orthogonal, parallel, or neither?

Find graphically and algebraically $2u + 3v$ and $u - v$

Find the angle between v and w

Find $u \cdot v$ (dot product), $u \times v$ (cross product), $u \cdot (v \times w)$, and $\|u\|$

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Panel 11

Planes / Lines:

Equation of line through $P(1,2,3)$ and $Q(4,5,1)$

Equation of plane through $P(1,0,2), Q(2,1,3), R(1,1,1)$

Intersection of lines / planes

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Panel 12

Distances

Find distance of $P(3,1)$ to line $2x - y = 1$

Find distance of $P(1,0,3)$ to plane $2x + 3y + z = 0$

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Panel 13

If $r(t) = < 4t, t^2, t^3 >$, find $r'(t)$, $r''(t)$, $\frac{d}{dt}\|r(t)\|$

If $r(t) = < e^t, 3t^3, \frac{3}{6t} >$ some curve, find $\int_1^2 r(t)dt$

If $r(t) = < t, \frac{1}{t} >$, find $T(t)$, $N(t)$, a_t and a_n

If $r(t) = \langle 3+t, 2t, 1-4t \rangle$, find N. Explain.

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Panel 14

If $r(t) = \langle 3-3t, 4t, t \rangle$ find length of curve as $t \in [0,1]$

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Panel 15

Find curvatue of $r(t) = \langle t, 3t^2, \frac{t^2}{2} \rangle$ at $t = 1$

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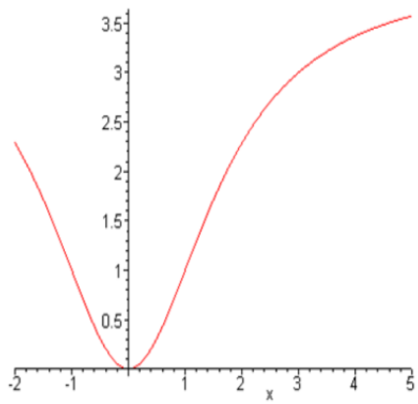
Panel 16

Picture Problems:

7. **Picture**: Sketch the circle that fits the graph below the best at the points x = 0 and x = 3. At which of the two points is the curvature smaller?



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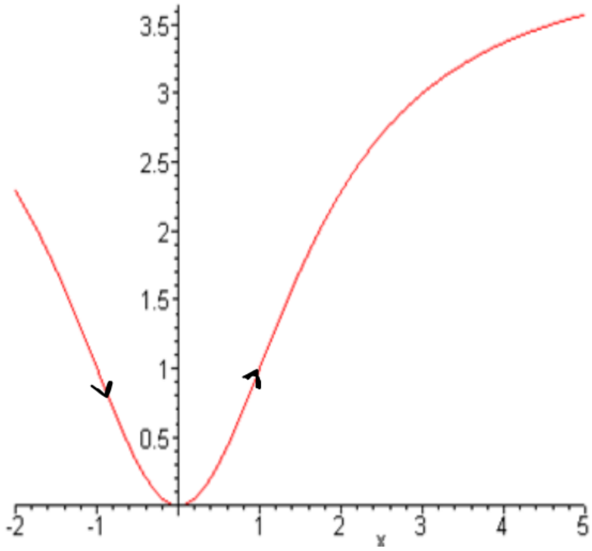
Panel 17

Picture Problems:

Sketch T, N, a, a_T, and a_N at t = 3



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Panel 18

Picture Problems

Match graphs to functions

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Panel 19

Story Problem:

What is the maximum height and range of a projectile fired at a height of 3 feet above the ground with an initial velocity of 900 feet/sec and at an angle of 45 degrees above the horizontal?

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Panel 20

Proofs:

Prove the following facts:

- Show that $u \times v = -(v \times u)$
- Show that $u \cdot (v \times u) = 0$
- Show that if $y = f(x)$ is a function that is twice continuously differentiable, then the curvature of *f* at a point *x* is $K = \dfrac{|f''(x)|}{\left(1 + [f'(x)]^2\right)^{3/2}}$
- Prove that the curvature of a line in space is zero.

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