Panel 1

Last time

Dot Product:

Cross Product:

Parametric Equation of line: $\ell(t) = P + t\vec{v}$

Scalar Equation of a plane: $ax+by+cz+d=0$

Extras: intersections between lines, lines + planes, planes, graphing planes

Distances: $\mathbb{R}^2$: $P(x_0,y_0)$ and $ax+by+c=0$: $d = \frac{|ax_0+by_0+c|}{\|\vec{n}\|}$, $n = \langle a,b \rangle$

$\mathbb{R}^3$: $P(x_0,y_0,z_0)$ and $ax+by+cz+d=0$: $d = \frac{|ax_0+by_0+cz_0+d|}{\|\vec{n}\|}$

$P(x_0,y_0,z_0)$ and line through Q, R: $d = \frac{\|\vec{PQ} \times \vec{PR}\|}{\|PQ\|}$

1

Panel 2

Quiz #3:

① Consider the line $\ell(t) = (1,3,-1) + t(2,1,-2)$ and the plane $2x+3y+z=6$

a) Find a point on the line (any point will do)

b) Find a point on the plane (any point will do)

c) Find the point where the line and plane intersect.

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Panel 3

② Consider the points P(1,2,3), Q(0,2,1), R(-1,2,1).

a) Find equation of the line through P and Q

b) Find equation of the plane through P, Q, and R.

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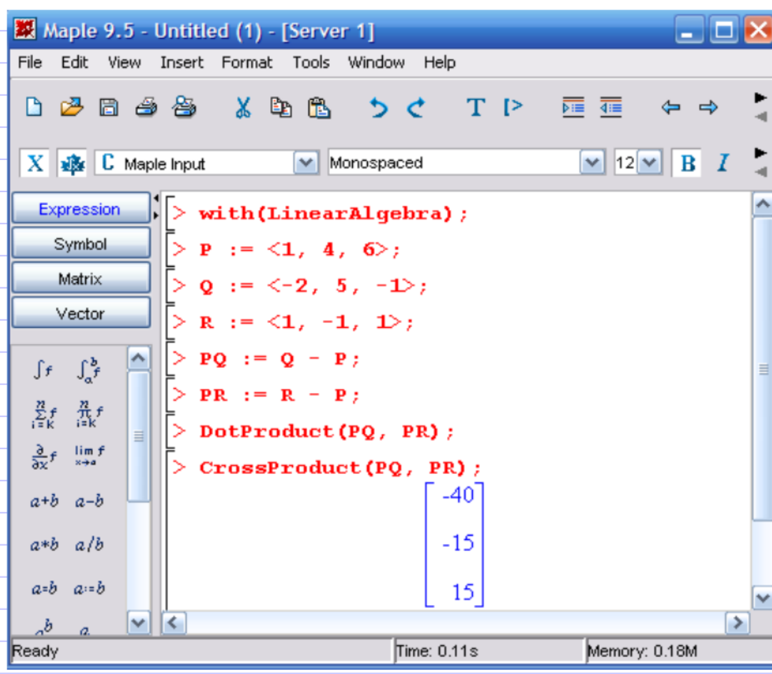
Panel 4

Maple: Dot + Cross Product



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Panel 5

Pop Quiz

Use Maple or compute manually:

(1) $\langle 1,3,9\rangle \cdot \langle 7,-2,-5\rangle$

a) 0 c) -44

b) 44 d) 12

(2) $\langle 1,3,9\rangle \times \langle 7,-2,-5\rangle$

a) 0 b) $\langle 3,-68,-23\rangle$

c) $\langle 1,2,3\rangle$ d) $\langle 3,68,-23\rangle$

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Panel 6

Section 12.6: Quadratic Surfaces

skip this

6

Panel 7

Chapter 12 Review

Started with $\mathbb{R}^3$, points, spheres, sheets $z^2+y^2=1$

Vectors: add, subtract, mult. by scalar, length

Dot product, angles, perp. vectors, projection

Cross product: perp. to 2 vectors, area of parallelogram

Lines ✓

Planes ✓

Distances ✓

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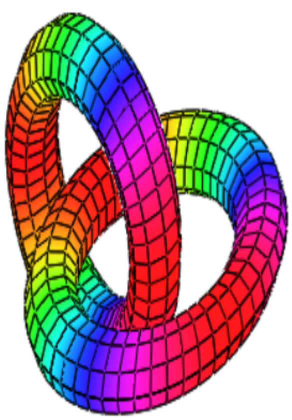
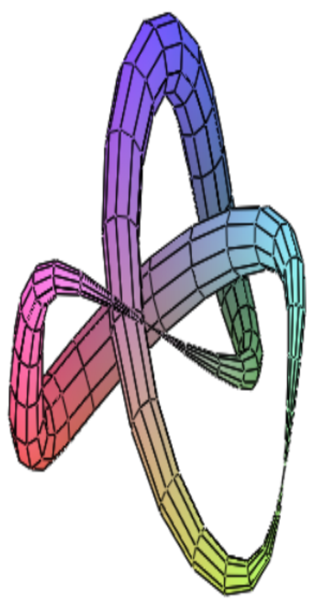
Panel 8

Space Curves





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Panel 9

Space Curves

Def: $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ is a vector-valued function with component functions f, g, and h

Many concepts work as they should: If $\vec{r}(t)$ is vector-valued function then

Limit: $\lim_{t \to t_0} \vec{r}(t) = \langle \lim_{t \to t_0} f(t), \lim_{t \to t_0} g(t), \lim_{t \to t_0} h(t) \rangle$

Derivative: $r'(t) = \langle f'(t), g'(t), h'(t) \rangle$

Integral: $\int \vec{r}(t)\,dt = \langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \rangle$

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Panel 10

The problem with vector-valued functions is to visualize them, and interprate the deriv. + integrals:

Ex: $\vec{r}(t) = \langle 1+t, 2+5t, -1+6t \rangle$ - describe graph

$= (1,2,-1) + t(1,5,6)$ is a line

Ex: $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$ - describe graph

Recall: $\langle \cos(t), \sin(t) \rangle$

$x = \cos(t)$, $y = \sin(t)$) $x^2 + y^2 = 1$ circles

Spiral around z-axis (slinky)

$\langle \sin(t), t, \cos(t) \rangle$

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Panel 11

Sketch graph of

$r_1(t) = \langle (4+\sin(20t))\cos(t), (4+\sin(20t))\sin(t), \cos(20t) \rangle$

$r_2(t) = \langle (2+\cos(1.5t))\cos t, (2+\cos(1.5t))\sin(t)\ \ \sin(1.5t) \rangle$

$z = x^2+y^2$

<u>Maple:</u> with(plots)

plot3d(x^2+y^2, x=-2..2, y=-2..2)

implicitplot3d($x^2+y^2+z^2=1$, x=-1..1, y=-1..1, z=-1..1),

spacecurve([x(t), y(t), z(t)])

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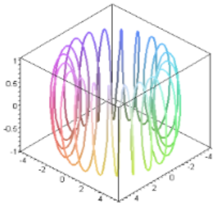
Panel 12

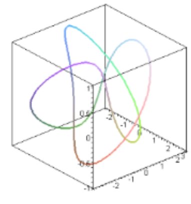
Sketch graph of

$r_1(t) = \langle (4+\sin(20t))\cos(t), (4+\sin(20t))\sin(t), \cos(20t) \rangle$

$r_2(t) = \langle (2+\cos(1.5t))\cos t, (2+\cos(1.5t))\sin(t)\ \ \sin(1.5t) \rangle$

```
> with(plots);
> spacecurve( [(4+sin(20*t))*cos(t), (4+sin(20*t))*sin(t), cos(20*t)], t=0..2*Pi, numpoints=500);
> spacecurve( [(2+cos(1.5*t))*cos(t), (2+cos(1.5*t))*sin(t), sin(1.5*t)], t=0..4*Pi, numpoints=500);
> spacecurve([t, t^2, t^3], t=0..2);
```





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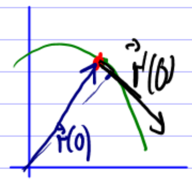
Panel 13

<u>Derivatives</u> of <u>Space Curves</u> aka Vector-valued functions

If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ and f, g, h are differentiable

then $\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$ (tangent vector to curve)



<u>Ex:</u> $\vec{r}(t) = \langle 1+t^3, te^{-t}, \sin(2t) \rangle$

Find $\vec{r}(0)$ and $\vec{r}\,'(0)$

Compute $\vec{r}(0) \cdot \vec{r}\,'(0)$

$\vec{r}\,'(t) = \langle 3t^2, e^{-t} - te^{-t}, 2\cos(2t) \rangle$

$\vec{r}(0) = \langle 1, 0, 0 \rangle$ $\quad \vec{r}\,'(0) = \langle 0, 1, 2 \rangle$ $\quad$ $t=0$

$\vec{r}(0) \cdot \vec{r}\,'(0) = 0$

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Panel 14

<u>Ex.</u> Find equation of tangent line to $\vec{r}(t) = \langle 2\cos t, \sin t, t \rangle$ at the point $P(0, 1, \pi/2)$

Need point and dir. vector

Point: $P(0, 1, \underline{\underline{\pi/2}})$ compare with $\langle 2\cos(t), \sin(t), \underline{\underline{t}} \rangle$

$(t = \pi/2)$

Direction of tangent: $\vec{r}\,'(t) = \langle -2\sin(t), \cos(t), 1 \rangle$

$\vec{r}\,'(\pi/2) = \langle -2, 0, 1 \rangle$

$\boxed{\ell(t) = (0, 1, \pi/2) + t\langle -2, 0, 1 \rangle}$

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Panel 15

Proof: $\frac{d}{dt}\left[\vec{u}(t)\cdot\vec{v}(t)\right] = \vec{u}'(t)\cdot\vec{v}(t) + \vec{u}(t)\cdot\vec{v}'(t)$ Thus

"Dot Product Rule"

HW $\mathbb{R}^2$

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Panel 16

Integrals of Space Curves aka vector valued functions

If $r(t) = \langle f(t), g(t), h(t)\rangle$ and f, g, h are integrable

then $\int \vec{r}(t)\,dt = \left\langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \right\rangle$

Ex: If $\vec{r}(t) = 2\cos(t)\,\vec{i} + \sin(t)\,\vec{j} + 2t\,\vec{k}$, find $\int_0^{\pi/2} \vec{r}(t)\,dt$

$$\int_0^{\pi/2} \vec{r}(t)\,dt = \left\langle \int_0^{\pi/2} 2\cos(t)\,dt, \int_0^{\pi/2} \sin(t)\,dt, \int_0^{\pi/2} 2t\,dt \right\rangle$$

$$= \left\langle 2\sin(t)\Big|_0^{\pi/2}, -\cos(t)\Big|_0^{\pi/2}, t^2\Big|_0^{\pi/2} \right\rangle =$$

$$= \left\langle 2, 1, \left(\tfrac{\pi}{2}\right)^2 \right\rangle$$

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