Panel 1

Last time $a\cdot a = \|a\|^2$

Dot Product: $a\cdot b = \|a\|\|b\|\cos\theta$

Cross Product: $\|a\times b\| = \|a\|\|b\|\sin\theta$, $a\times b \perp a, b$

Parametric Equation of line:

$$\ell(t) = P_0 + t\vec{v} = \langle x_0+tv_1, y_0+tv_2, z_0+tv_3\rangle$$

Scalar Equation of a plane:

$$ax+by+cz+d=0 \quad , (a,b,c)=\vec{n}$$

(or $p(s,t) = P_0 + t\vec{v} + s\vec{w}$)

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Panel 2

http://www.slideshare.net/leingang/lesson-4-lines-planes-and-the-distance-formula

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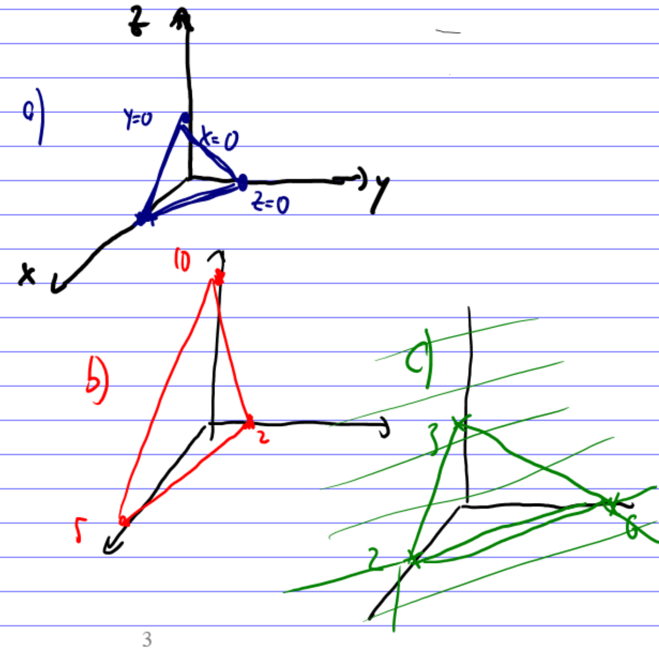
Panel 3

Graphing Planes: Planes can be visualized by looking at the traces in the coordinate planes.

a) $x + y + z = 1$



$2x + 5y + z = 10$

b)

$3x + y + 2z = 6$

c)

Panel 4

Intersections:

Find intersection of $\ell(t) = \langle 1,2,3\rangle + t\langle 1,1,1\rangle$ and
$2x - y + z = 0$

Find intersection of $\ell(t) = \langle 0,1,0\rangle + t\langle 1,0,1\rangle$ and
$\ell(s) = \langle -1,-2,1\rangle + t\langle 2,3,0\rangle$

Find intersection of $-x + y + z = 0$ and
$2x - y + z = 1$

Panel 5

Find intersection of $\ell(t) = \langle 1,2,3\rangle + t\langle 1,1,1\rangle$ and
$2x - y + z = 0$

Directional vector of line is $\langle 1,1,1\rangle$, $\vec{n}$ to plane is $\langle 2,-1,1\rangle$
$\langle 1,1,1\rangle \cdot \langle 2,-1,1\rangle = 2$ are not perp. so

They intersect. Is $P(0,1,2)$ part of the plane? No.

$\ell(t) = \langle \underbrace{1+t}_{x}, \underbrace{2+t}_{y}, \underbrace{3+t}_{z}\rangle$: $2(1+t) - (2+t) + (3+t) = 0$
$\cancel{2} + 2t - \cancel{2} - \cancel{t} + 3 + \cancel{t} = 0$
$3 + 2t = 0$, $t = -\frac{3}{2}$

$Q = \ell(-\frac{3}{2}) = \langle 1,2,3\rangle - \langle \frac{3}{2}, \frac{3}{2}, \frac{3}{2}\rangle = \langle -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\rangle$

Panel 6

Find intersection of $\ell_1(t) = \langle 0,1,0\rangle + t\langle 1,0,1\rangle$ and
$\ell_2(s) = \langle -1,-2,1\rangle + t\langle 2,3,0\rangle$

$\ell_1(t) = \ell_2(s) \iff$
$0 + t = -1 + 2s$ $\quad t = -1 + 2s$
$1 + 0t = -2 + 3s$ $\quad 1 = -2 + 3s$ (2)
$0 + 1t = 1 + 0s$ $\quad t = 1$ (3)

(2) $t = 1, s = 1 \Rightarrow$ Now check 1st equation: $1 = -1 + 2$ ✓

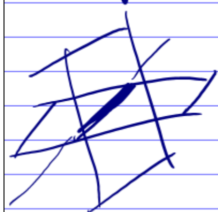
Yes: point of intersection is $\ell_1(1) = \langle 1,1,1\rangle$

Panel 7

Find intersection of $-x+y+z=0$ and $2x-y+z=1$

$\vec{n}_1 = \langle -1,1,1\rangle$, $\vec{n}_2 = \langle 2,-1,1\rangle$ are not parallel!

They intersect in a line, i.e. need point, direction



$z=0$: line goes through xy plane:

$$\begin{aligned} -x+y&=0 \\ 2x-y&=1 \end{aligned} \Rightarrow x=1,\ y=1$$

Point on line is $P(1,1,0)$. Directional vector of line is orp. to normal vectors of both planes. $\Rightarrow$ It is parallel to $\vec{n}_1 \times \vec{n}_2$

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Panel 8

$$\begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = \langle 2,3,-1\rangle$$

$$\ell(t) = \langle 1,1,0\rangle + t\langle 2,3,-1\rangle$$

$-x+y+z=0$, $2x-y+z=1$

$-(1+2t)+(1+3t)+(-t)=0$ $\quad 2(1+2t)-(1+3t)+(-t)=1$ ✓

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Panel 9

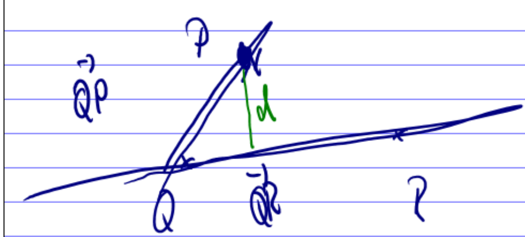
$x+y+z=0$ and $2x-y+z=1$

Panel 10

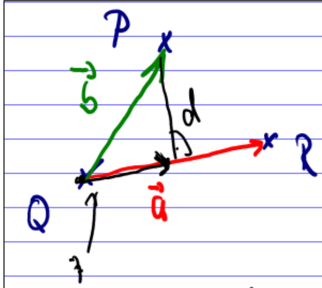
Distance of point $P(x_0,y_0)$ from line $ax+by+c=0$ in $\mathbb{R}^2$:

$$\frac{|ax_0+by_0+c|}{\|\vec{n}\|}, \quad \vec{n}=\langle a,b\rangle$$

Distance of point $P(x_0,y_0,z_0)$ from line through R and Q in $\mathbb{R}^3$:



Panel 11



Hint: $a\times(b\times \underset{a}{c}) = (a\cdot \underset{a}{c})b - (a\cdot b)\underset{a}{c}$

$\vec r = \text{proj}_{\vec a}(\vec b)$, $d = \|\vec b - \text{proj}_{\vec a}(\vec b)\|$

$$\left\|\vec b - \frac{a\cdot b}{\|a\|^2}a\right\| = \left\|\frac{\|a\|^2 b - (a\cdot b)a}{\|a\|^2}\right\| = \frac{1}{\|a\|^2}\left\|\,\|a\|^2 b - (a\cdot b)a\right\|$$

$$= \frac{1}{\|a\|^2}\|(a\cdot a)b - (a\cdot b)a\| = \frac{1}{\|a\|^2}\|a\times(b\times a)\| =$$

$$= \frac{1}{\|a\|^2}\|a\|\,\|b\times a\|\,|\sin\underset{\pi/2}{\theta}| = \frac{1}{\|a\|^{\cancel{2}}}\cancel{\|a\|}\,\|b\times a\|$$

$$d = \frac{\|a\times b\|}{\|a\|}$$

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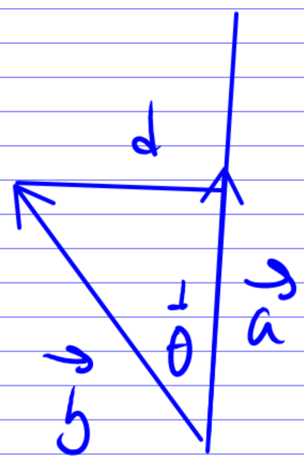
Panel 12



$$\sin\theta = \frac{d}{\|b\|}$$

$$\frac{\|a\|\,\|b\|\sin\theta}{\|a\|} = d$$

$$\frac{\|a\times b\|}{\|a\|} = d$$

very good.

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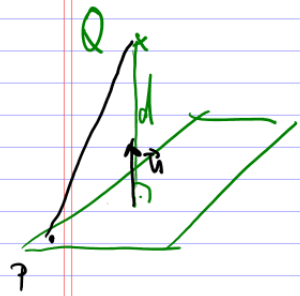
Panel 13

Distance between Point $Q(x_0, y_0, z_0)$ and Plane



$$ax + by + cz + d = 0$$

Find $P(x_1, y_1, z_1)$ on the plane

$$d = \|\text{proj}_{\vec{n}} \vec{PQ}\| =$$

$$= \frac{\vec{PQ} \cdot \vec{n}}{\|n\|} = \frac{|\langle x_0 - x_1, y_0 - y_1, z_0 - z_1\rangle \langle a, b, c\rangle|}{\|\vec{n}\|}$$

$$= \frac{|a(x_0 - x_1) + b(y_0 - y_1) + c(z_0 - z_1)|}{\|\vec{n}\|} = \frac{|ax_0 + by_0 + cz_0 - ax_1 - by_1 - cz_1|}{\|\vec{n}\|} =$$

$$= \frac{|ax_0 + by_0 + cz_0 + d|}{\|\vec{n}\|}$$

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Panel 14

Distance Formulas

$P(x_0, y_0)$ and line $ax + by + c = 0$ in $\mathbb{R}^2$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$P(x_0, y_0, z_0)$ and plane $ax + by + cz + d$ in $\mathbb{R}^3$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$P(x_0, y_0, z_0)$ and line through Q, R in $\mathbb{R}^3$

$$d = \frac{\|\vec{PQ} \times \vec{PR}\|}{\|QR\|} = \frac{\|a \times b\|}{\|a\|}$$

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Panel 15

Find distance between

a) $10x + 2y - 2z = 5$ and $x + y + z = 1$

b) $10x + 2y - 2z = 5$ and $5x + y - z = 1$

a) Planes are not parallel, so they intersect, distance is zero

b) Pick any point on 2^{nd} plane, e.g. $(0,0,-1)$ or $(0,1,0)$

Use prev. formula!

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